Developments and Applications of Adaptive Cerebellar Model Articulation Controller

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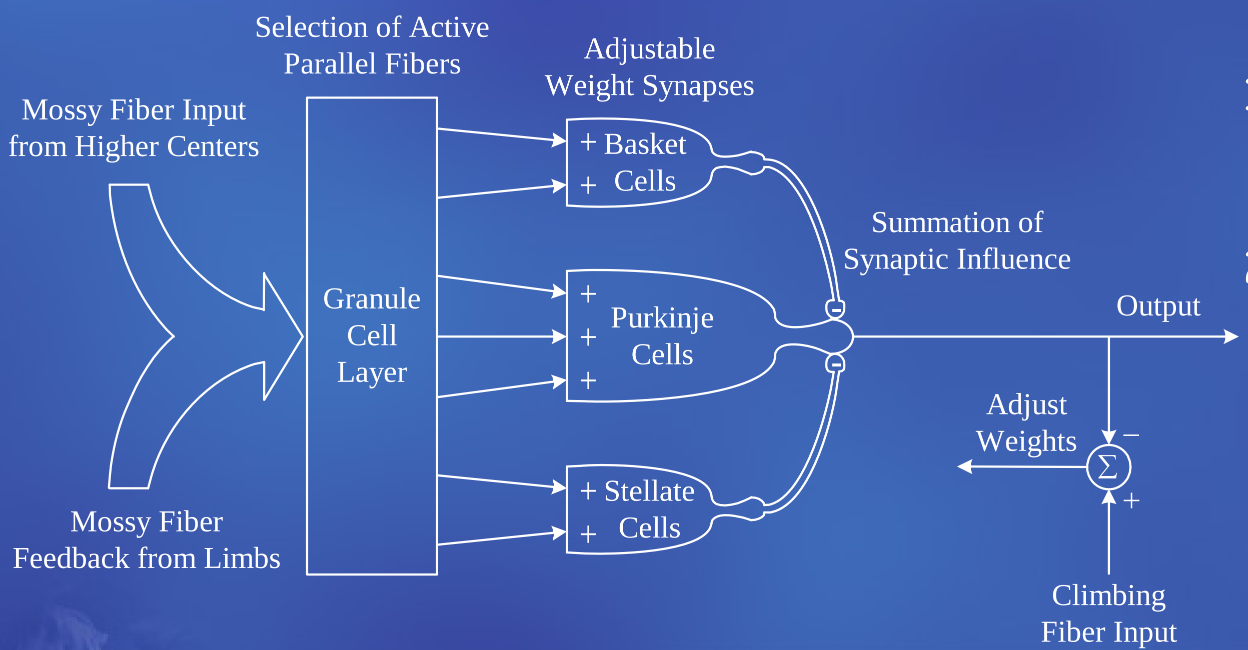
Referred Papers

1. Introduction of Cerebellar Model Articulation Controller (CMAC)

The Cerebellar Cortex

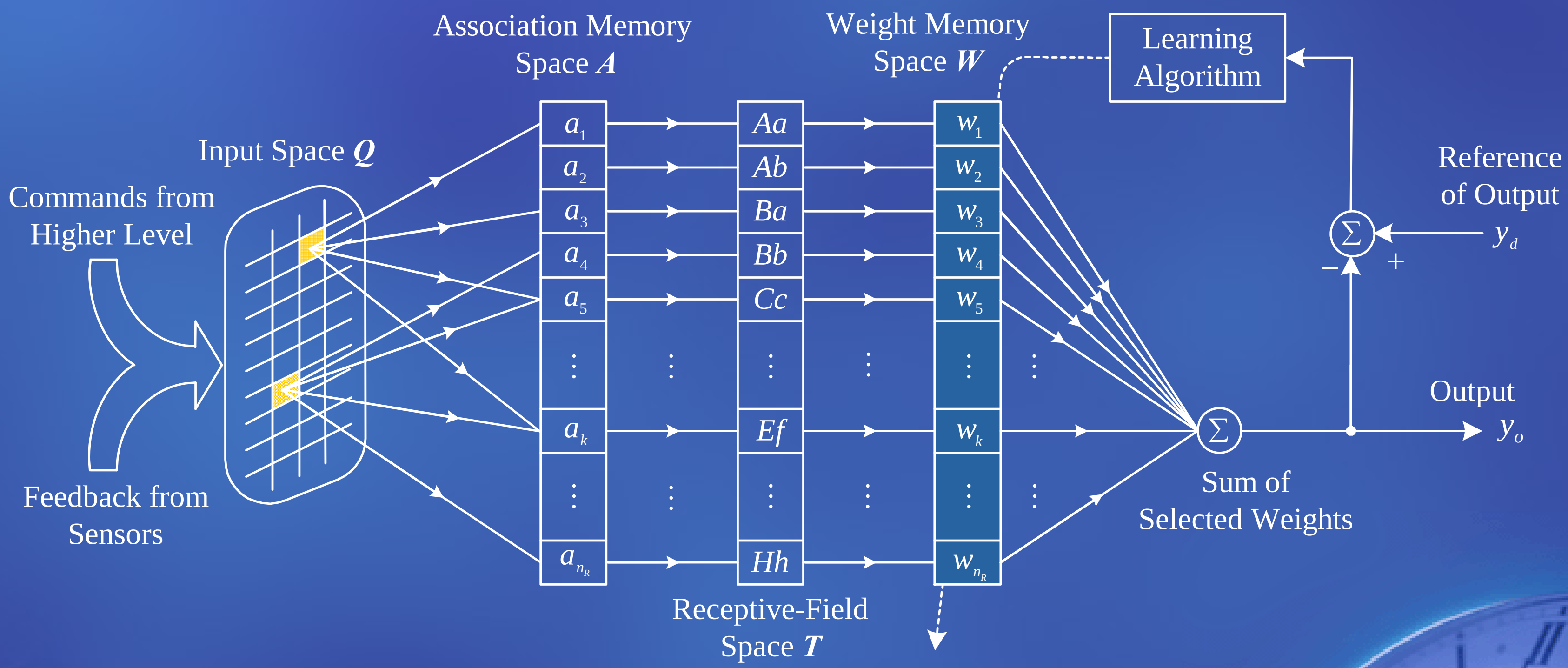
- A model of the cerebellar cortex (1969 Marr)



1. Information is stored in overlap layers
2. Quickly recall of the stored information

Original Cerebellar Model Articulation Controller

- The basic concept of an original CMAC (Albus 1971).



- Mapping $Q \rightarrow A$: Transforms the input vector q into an association memory selection vector $a(q) \in A$.

$$a(q) = [a_1(q), a_2(q), \cdots, a_k(q), \cdots, a_{n_R}(q)]^T \quad (1.1)$$

- Mapping $A \rightarrow T$: Each location of A corresponds to a receptive-field (binary receptive-field).
- Mapping $T \rightarrow W$:

$$w = [w_1, w_2, \cdots, w_k, \cdots, w_{n_R}]^T \quad (1.2)$$

- Output computation y_o :

$$y_o(q) = a^T(q)w = \sum_{k=1}^{n_R} a_k(q)\, w_k \quad (1.3)$$

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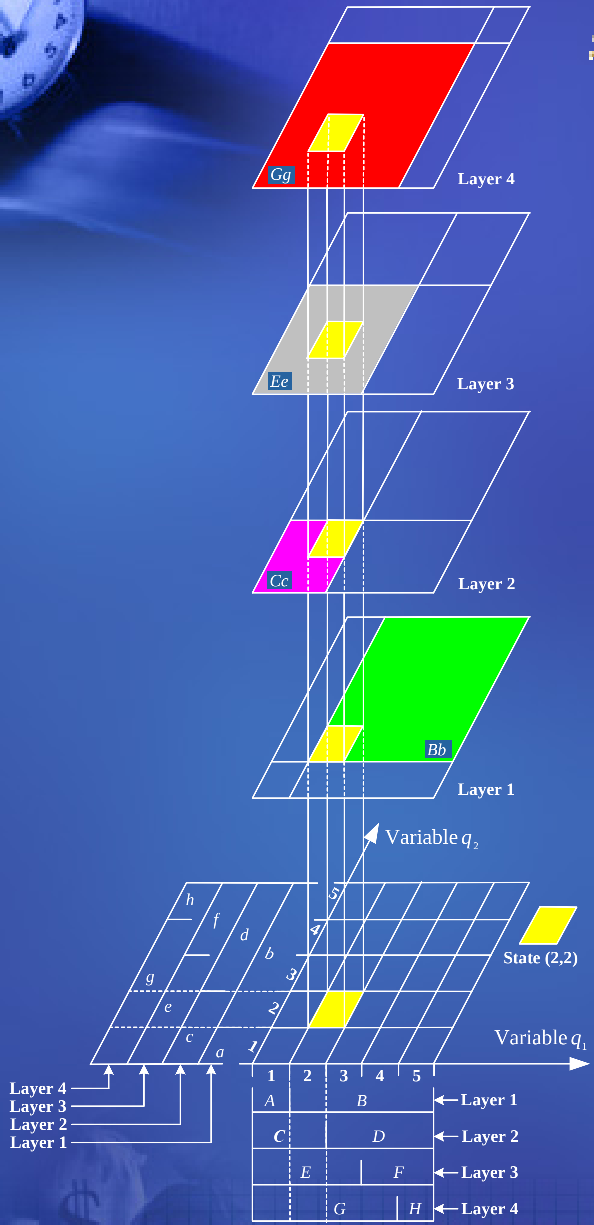
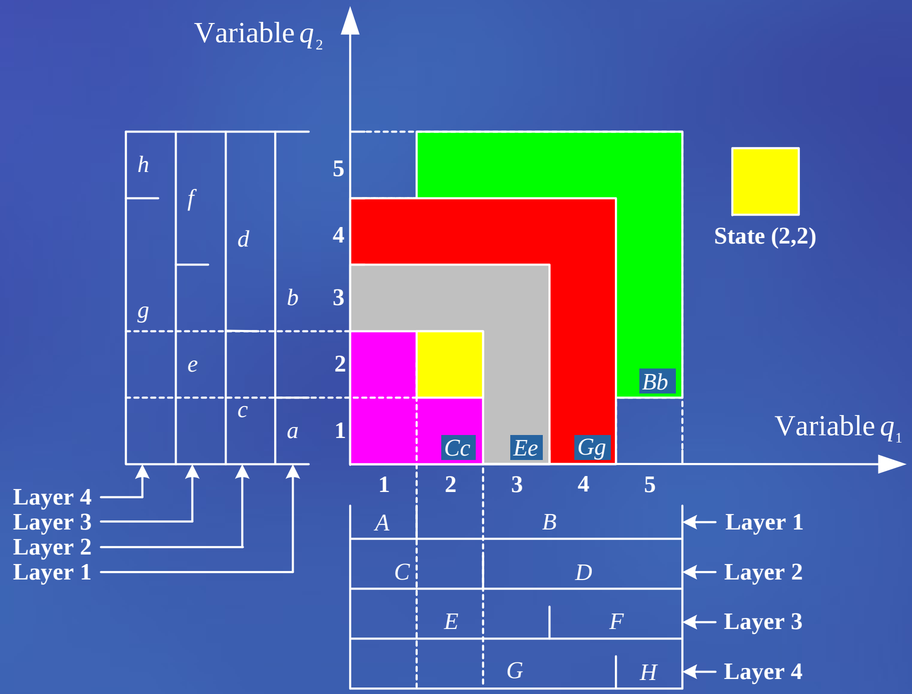
The schematic representation of a 2-D CMAC. (binary receptive-fields)

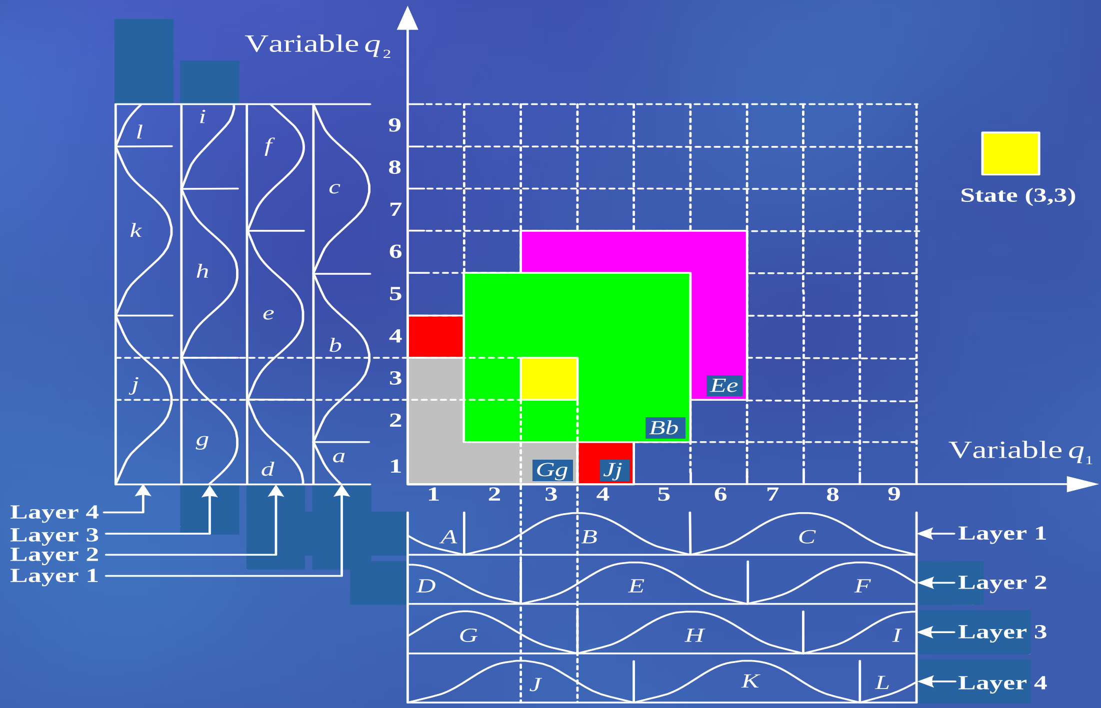




Non-differentiable receptive-fields
Non-smooth function approximation
No stability analysis

General Cerebellar Model Articulation Controller

- The schematic diagram of a general 2-D CMAC.



Differentiable receptive-fields
Smooth function approximation
Stability analysis

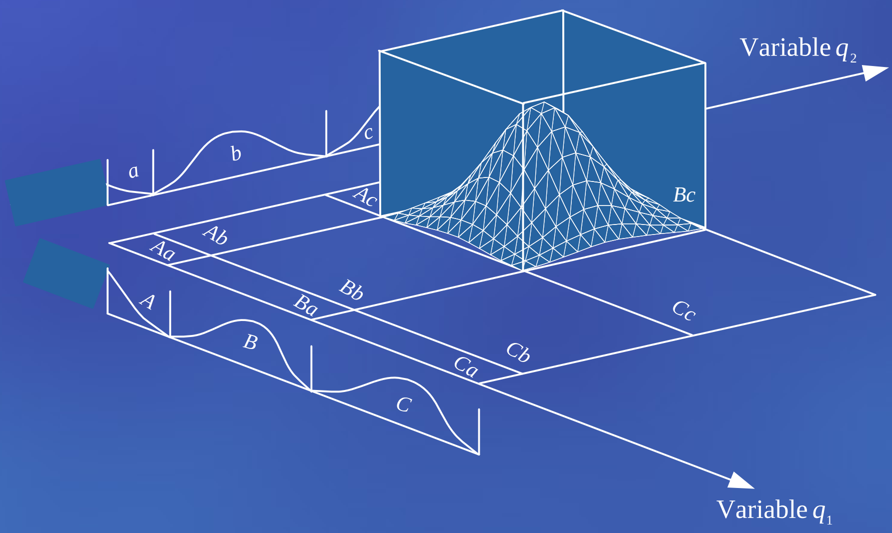
- *Gaussian receptive-field basis function*:

$$\phi_{ik}(q_i) = \exp\left[\frac{-(q_i - m_{ik})^2}{v_{ik}^2}\right] \quad (1.4)$$

- *Multidimensional receptive-field function:*

$$b_k(\boldsymbol{q}, \boldsymbol{m}_k, \boldsymbol{v}_k) = \prod_{i=1}^{n} \phi_{ik}(q_i) = \exp\left[\sum_{i=1}^{n} \frac{-(q_i - m_{ik})^2}{v_{ik}^2}\right] \quad (1.5)$$

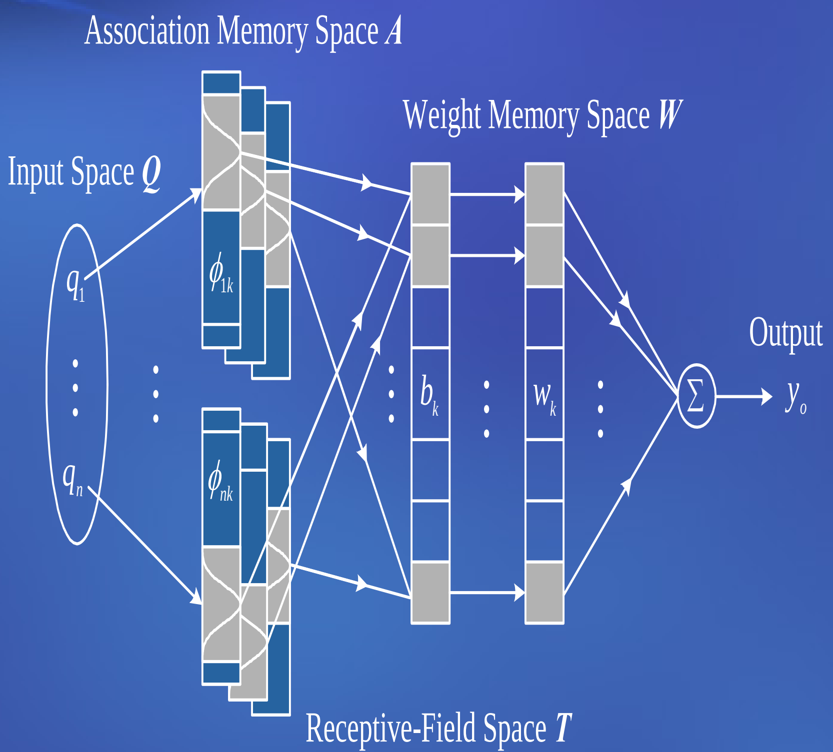
- The receptive-field Bc with including a Gaussian receptive-field basis function.



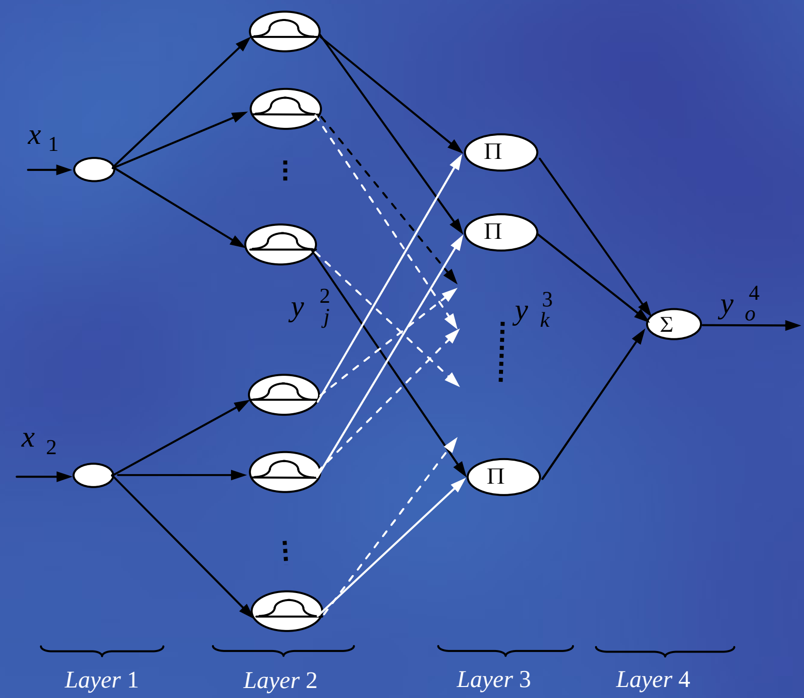
- The output of CMAC

$$y_o = \boldsymbol{w}^T \Gamma(\boldsymbol{q}, \boldsymbol{m}, \boldsymbol{v}) = \sum_{k=1}^{n_R} w_k\, b_k(\boldsymbol{q}, \boldsymbol{m}_k, \boldsymbol{v}_k) \tag{1.6}$$

CMAC



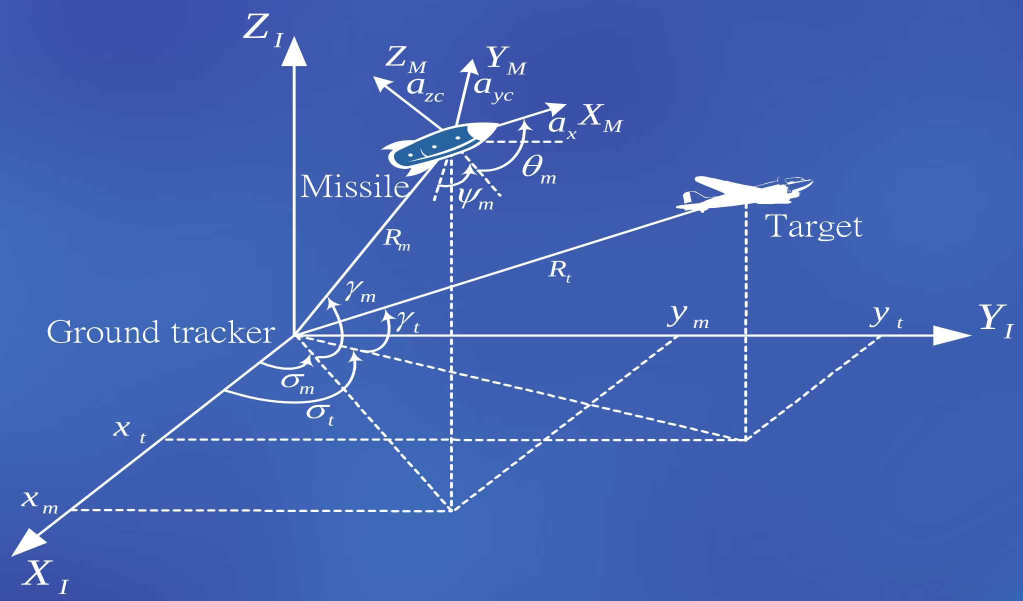
Neural Network

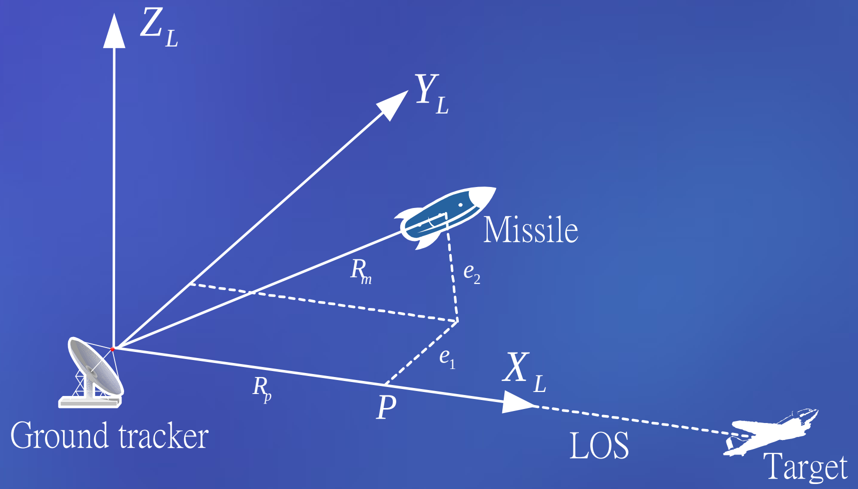


Good generalization capability
Less computation
Better approximation ability

2. Missile Guidance Law Design Using CMAC

- Formulation of Missile-Target Engagement
  - The 3-D missile-target pursuit diagram.





- The motion of the missile in the inertial frame.

$$
\begin{aligned}
\ddot{x}_m &= a_x \cos\theta_m \cos\psi_m - a_{yc}(\sin\phi_{mc}\sin\theta_m\cos\psi_m + \cos\phi_{mc}\sin\psi_m) \\
&\quad - a_{zc}(\cos\phi_{mc}\sin\theta_m\cos\psi_m - \sin\phi_{mc}\sin\psi_m) \\
\ddot{y}_m &= a_x \cos\theta_m \sin\psi_m - a_{yc}(\sin\phi_{mc}\sin\theta_m\sin\psi_m - \cos\phi_{mc}\cos\psi_m) \\
&\quad - a_{zc}(\cos\phi_{mc}\sin\theta_m\sin\psi_m + \sin\phi_{mc}\cos\psi_m) \\
\ddot{z}_m &= a_x \sin\theta_m + a_{yc}\sin\phi_{mc}\cos\theta_m + a_{zc}\cos\phi_{mc}\cos\theta_m - g_v \\
\dot{\psi}_m &= a_{yc}\cos\phi_{mc}/(v_m\cos\theta_m) + a_{zc}\sin\phi_{mc}/(v_m\cos\theta_m) \\
\dot{\theta}_m &= a_{yc}\sin\phi_{mc}/v_m + a_{zc}\cos\phi_{mc}/v_m - g_v\cos\theta_m/v_m
\end{aligned}
\tag{2.1}
$$

- The missile position in the LOS frame.

$$\begin{bmatrix} e_1 \\ e_2 \end{bmatrix} = \begin{bmatrix} -\sin(\sigma_m - \sigma_t) & \cos(\sigma_m - \sigma_t) & 0 \\ -\sin(\gamma_m - \gamma_t)\cos(\sigma_m - \sigma_t) & -\sin(\gamma_m - \gamma_t)\sin(\sigma_m - \sigma_t) & \cos(\gamma_m - \gamma_t) \end{bmatrix} \begin{bmatrix} x_m \\ y_m \\ z_m \end{bmatrix} \quad (2.2)$$

- Define

$$\boldsymbol{x} \triangleq [x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8]^T$$
$$\triangleq [x_m, y_m, z_m, \dot{x}_m, \dot{y}_m, \dot{z}_m, \psi_m, \theta_m]^T$$
$$\boldsymbol{u} \triangleq [u_1, u_2]^T \triangleq [a_{yc}, a_{zc}]^T \tag{2.3}$$

- The tracking error dynamic equation

$$\begin{bmatrix} \ddot{e}_1 \\ \ddot{e}_2 \end{bmatrix} = \begin{bmatrix} F_1(\boldsymbol{x},t) \\ F_2(\boldsymbol{x},t) \end{bmatrix} + \begin{bmatrix} G_{11}(\boldsymbol{x},t) & G_{12}(\boldsymbol{x},t) \\ G_{21}(\boldsymbol{x},t) & G_{22}(\boldsymbol{x},t) \end{bmatrix} \boldsymbol{u}(t)$$
$$\triangleq \boldsymbol{F}(\boldsymbol{x},t) + \boldsymbol{G}(\boldsymbol{x},t)\boldsymbol{u}(t) \tag{2.4}$$

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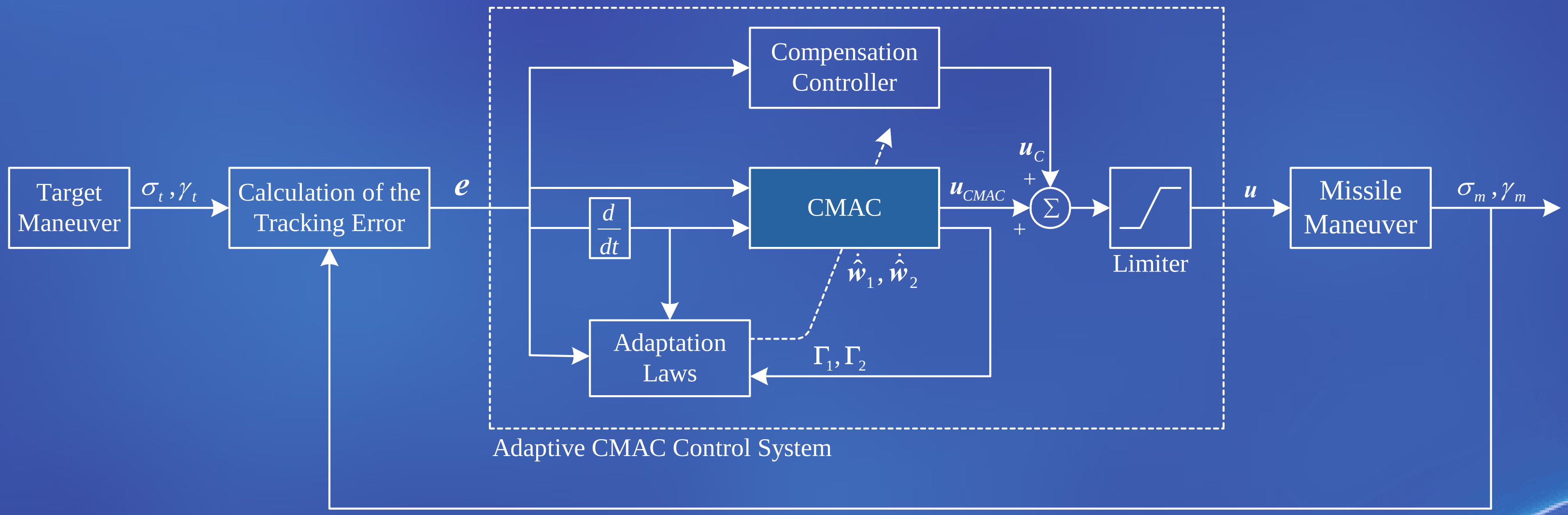
- The CMAC control system.

The control law:

$$\mathbf{u} = \mathbf{u}_{CMAC} + \mathbf{u}_{C} \qquad (2.5)$$



- A feedback linearization control law

$$u_T = G^{-1}(x,t)[-F(x,t) - K_v\dot{e} - K_p e] \tag{2.6}$$

Substituting (2.6) into (2.4), yields

$$\ddot{e} + K_v\dot{e} + K_p e = 0 \tag{2.7}$$

- The minimum approximation error

$$\varepsilon_j = u_{CMACj}(q_j, w_j^*) - u_{Tj} \tag{2.8}$$

- CMAC control system

$$u_T = u_{CMAC}(q,\hat{w}) + u_C(q) = \hat{w}^T\Gamma(q) + u_C(q) \tag{2.9}$$

Error equation

$$\ddot{e} = -K_v\dot{e} - K_p e + G(x,t)[u_{CMAC} + u_C - u_T] \tag{2.10}$$

- The error dynamics in the state-space form

$$\begin{aligned}\dot{\boldsymbol{E}} &= \boldsymbol{\Lambda}\boldsymbol{E} + \boldsymbol{G}_m(\boldsymbol{x},t)[\boldsymbol{u}_{CMAC} + \boldsymbol{u}_C - \boldsymbol{u}_T] \\ &= \boldsymbol{\Lambda}\boldsymbol{E} + \boldsymbol{G}_{m1}(\boldsymbol{x},t)[u_{CMAC1} + u_{C1} - u_{T1}] \\ &\quad + \boldsymbol{G}_{m2}(\boldsymbol{x},t)[u_{CMAC2} + u_{C2} - u_{T2}]\end{aligned} \tag{2.11}$$

where

$$\boldsymbol{E} \triangleq [e_1, \dot{e}_1, e_2, \dot{e}_2]^T$$

$$\boldsymbol{G}_m(\boldsymbol{x},t) = [\boldsymbol{G}_{m1}(\boldsymbol{x},t), \quad \boldsymbol{G}_{m2}(\boldsymbol{x},t)] = \begin{bmatrix} 0 & 0 \\ G_{11}(\boldsymbol{x},t) & G_{12}(\boldsymbol{x},t) \\ 0 & 0 \\ G_{21}(\boldsymbol{x},t) & G_{22}(\boldsymbol{x},t) \end{bmatrix}$$

$$\boldsymbol{\Lambda} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -k_{p1} & -k_{v1} & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -k_{p2} & -k_{v2} \end{bmatrix}$$

Theorem 2.1: If the control law is designed as (2.5), in which the adaptation law of CMAC

$$\dot{\hat{\boldsymbol{w}}}_j = \dot{\tilde{\boldsymbol{w}}}_j = -\beta_{wj}\boldsymbol{E}^T\boldsymbol{P}\boldsymbol{G}_{mj}\Gamma_j \qquad (2.12)$$

and the compensation control is designed as

$$u_{Cj} = -\delta_j \operatorname{sgn}(\boldsymbol{E}^T\boldsymbol{P}\boldsymbol{G}_{mj}) \qquad (2.13)$$

then the stability of the guidance system can be guaranteed.

- *Proof:* A Lyapunov function is defined as

$$V(t)=\frac{1}{2}E^{T}PE+\frac{1}{2\beta_{w1}}\tilde{w}_1^{T}\tilde{w}_1+\frac{1}{2\beta_{w2}}\tilde{w}_2^{T}\tilde{w}_2 \quad (2.14)$$

The Lyapunov equation

$$\Lambda^{T}P+P\Lambda=-R \quad (2.15)$$

Taking the derivative of the Lyapunov function.

$$\begin{aligned}\dot{V}(t)=&-\frac{1}{2}\mathbf{E}^{T}\mathbf{R}\mathbf{E}+\frac{1}{\beta_{w1}}\tilde{\mathbf{w}}_1^{T}\dot{\tilde{\mathbf{w}}}_1+\frac{1}{\beta_{w2}}\tilde{\mathbf{w}}_2^{T}\dot{\tilde{\mathbf{w}}}_2\\&+\mathbf{E}^{T}\mathbf{P}\mathbf{G}_{m1}[u_{C1}+\varepsilon_1]+\mathbf{E}^{T}\mathbf{P}\mathbf{G}_{m1}\tilde{\mathbf{w}}_1^{T}\Gamma_1\\&+\mathbf{E}^{T}\mathbf{P}\mathbf{G}_{m2}[u_{C2}+\varepsilon_2]+\mathbf{E}^{T}\mathbf{P}\mathbf{G}_{m2}\tilde{\mathbf{w}}_2^{T}\Gamma_2\end{aligned} \quad (2.16)$$

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Choosing (2.12), then (2.16) can be simplified

$$\dot{V}(t) \le -\frac{1}{2}\mathbf{E}^T\mathbf{R}\mathbf{E} + \mathbf{E}^T\mathbf{P}\mathbf{G}_{m1}u_{C1} + \left|\mathbf{E}^T\mathbf{P}\mathbf{G}_{m1}\right| |\varepsilon_1| + \mathbf{E}^T\mathbf{P}\mathbf{G}_{m2}u_{C2} + \left|\mathbf{E}^T\mathbf{P}\mathbf{G}_{m2}\right| |\varepsilon_2| \quad (2.17)$$

Setting (2.13), then (2.17) can be rewritten as

$$\begin{aligned}\dot{V}(t) &\le \frac{1}{2}\mathbf{E}^T\mathbf{R}\mathbf{E} + (|\varepsilon_1| - \delta_1)\left|\mathbf{E}^T\mathbf{P}\mathbf{G}_{m1}\right| + (|\varepsilon_2| - \delta_2)\left|\mathbf{E}^T\mathbf{P}\mathbf{G}_{m2}\right| \\ &\le -\frac{1}{2}\mathbf{E}^T\mathbf{R}\mathbf{E} \le 0\end{aligned} \quad (2.18)$$

Numerical Simulations

- The target motion model.

$$\ddot{x}_t = -a_{ty}\sin\psi_t - a_{tz}\sin\theta_t\cos\psi_t$$

$$\ddot{y}_t = a_{ty}\cos\psi_t - a_{tz}\sin\theta_t\sin\psi_t$$

$$\ddot{z}_t = a_{tz}\cos\theta_t - g_v$$

$$\dot{\psi}_t = a_{ty}/(v_t\cos\theta_t)$$

$$\dot{\theta}_t = (a_{tz} - g_v\cos\theta_t)/v_t \qquad (2.19)$$

- For scenarios 1 and 2:

  $a_{ty} = 5g_v$ $\quad a_{tz} = -g_v$ for the first 2.5 sec

  $a_{ty} = -5g_v$ $\quad a_{tz} = 5g_v$ until interception

- For scenarios 3:

  $a_{tz} = -g_v$ $\quad a_{ty} = 0g_v$ for the first 2.5 sec

  $a_{ty} = 0.5g_v$ $\quad a_{tz} = -g_v$ until interception

- The feedback linearization guidance law

$$\boldsymbol{u}_T = \boldsymbol{G}^{-1}(\boldsymbol{x},t)[-\boldsymbol{F}(\boldsymbol{x},t) - \boldsymbol{K}_v\dot{\boldsymbol{e}} - \boldsymbol{K}_p\boldsymbol{e}]$$

where $\boldsymbol{K}_v = \begin{bmatrix} 14 & 0 \\ 0 & 14 \end{bmatrix}$, $\boldsymbol{K}_p = \begin{bmatrix} 49 & 0 \\ 0 & 49 \end{bmatrix}$ (2. 20)

- Adaptive CMAC-based guidance law

The design parameters are set as follows:

$$\boldsymbol{R} = \begin{bmatrix} 588 & 83 & 0 & 0 \\ 83 & 16 & 0 & 0 \\ 0 & 0 & 588 & 83 \\ 0 & 0 & 83 & 16 \end{bmatrix}, \ \beta_{w1} = \beta_{w2} = 15, \ \delta_1 = \delta_2 = 0.01 \qquad (2.21)$$

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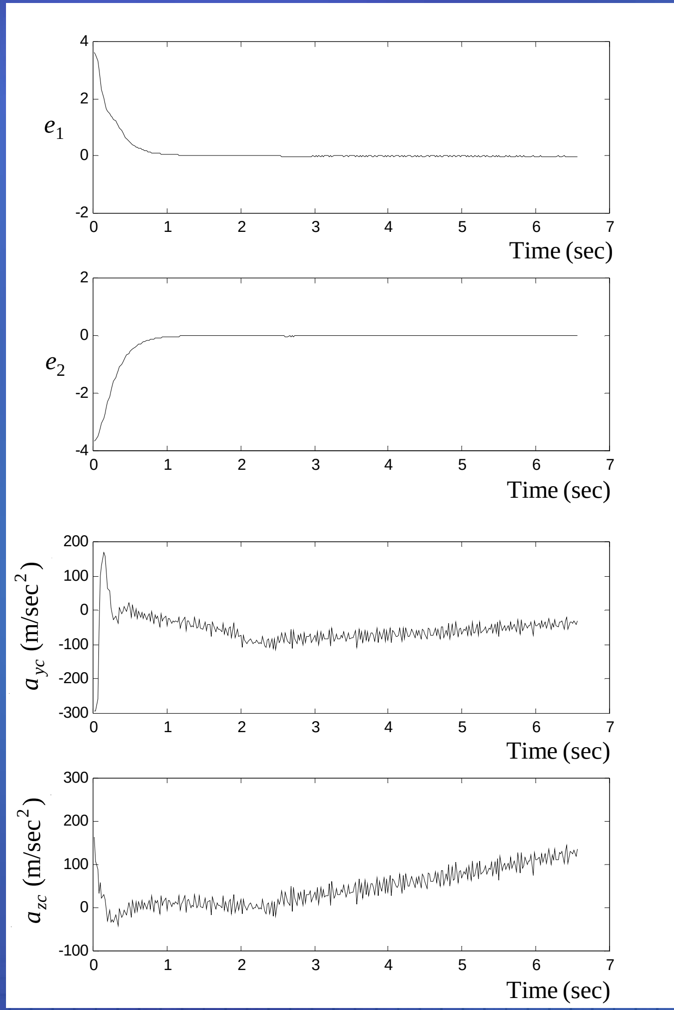
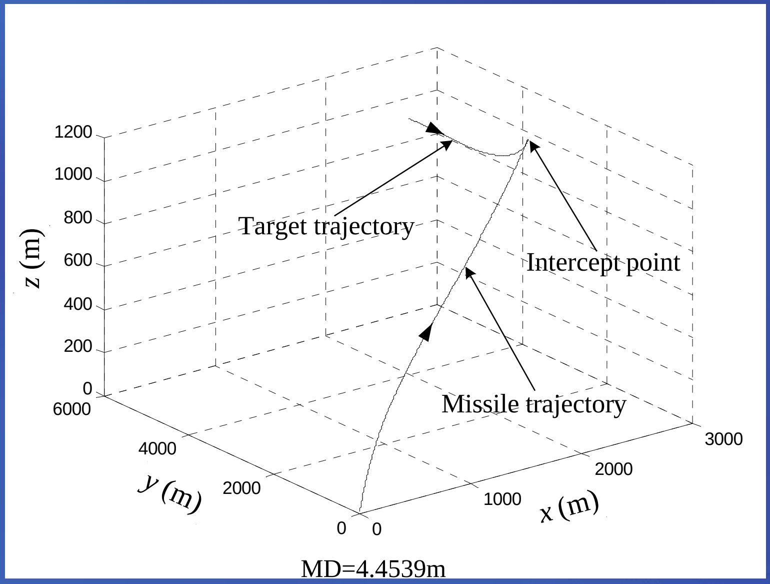
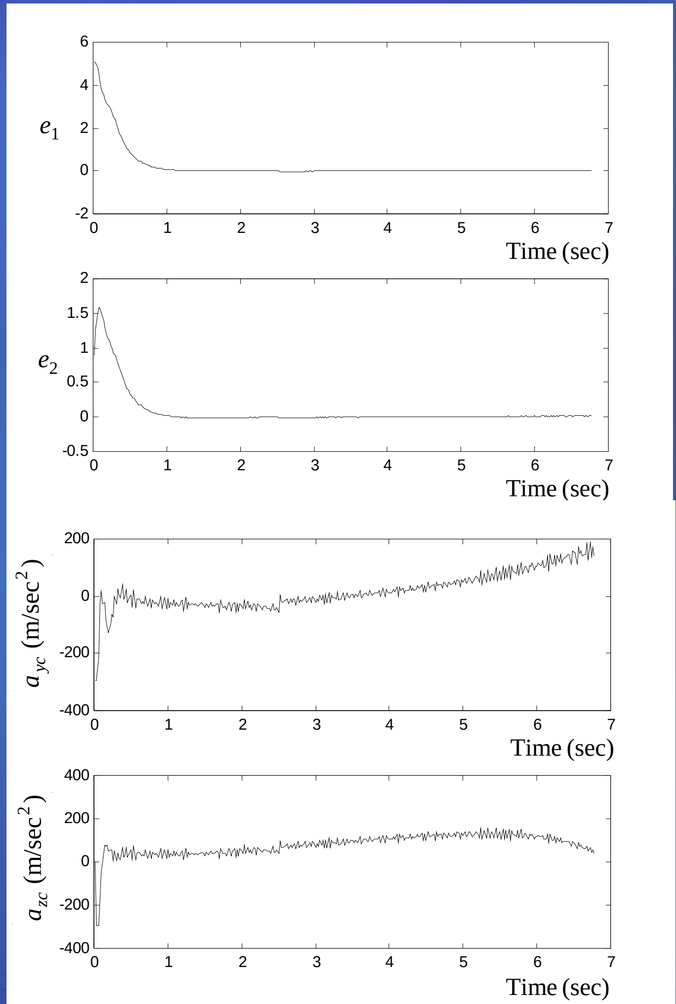
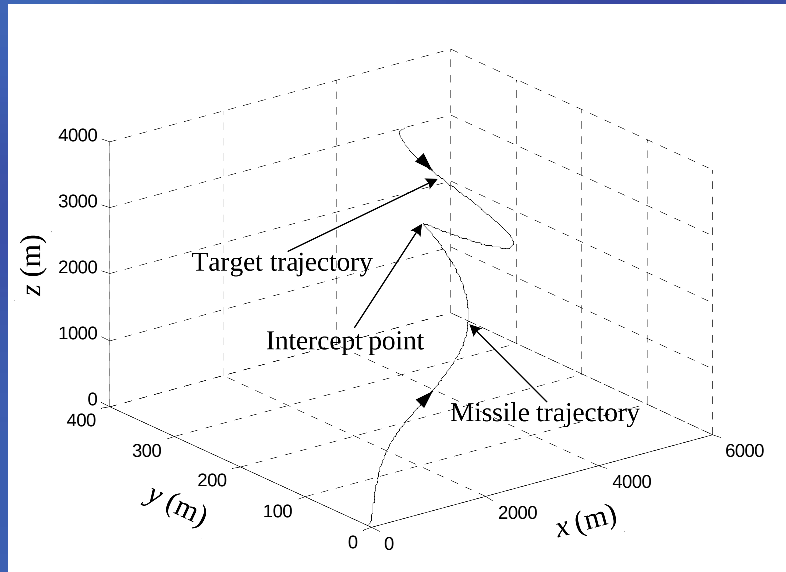
Engagement scenario 1 with feedback linearization guidance law.





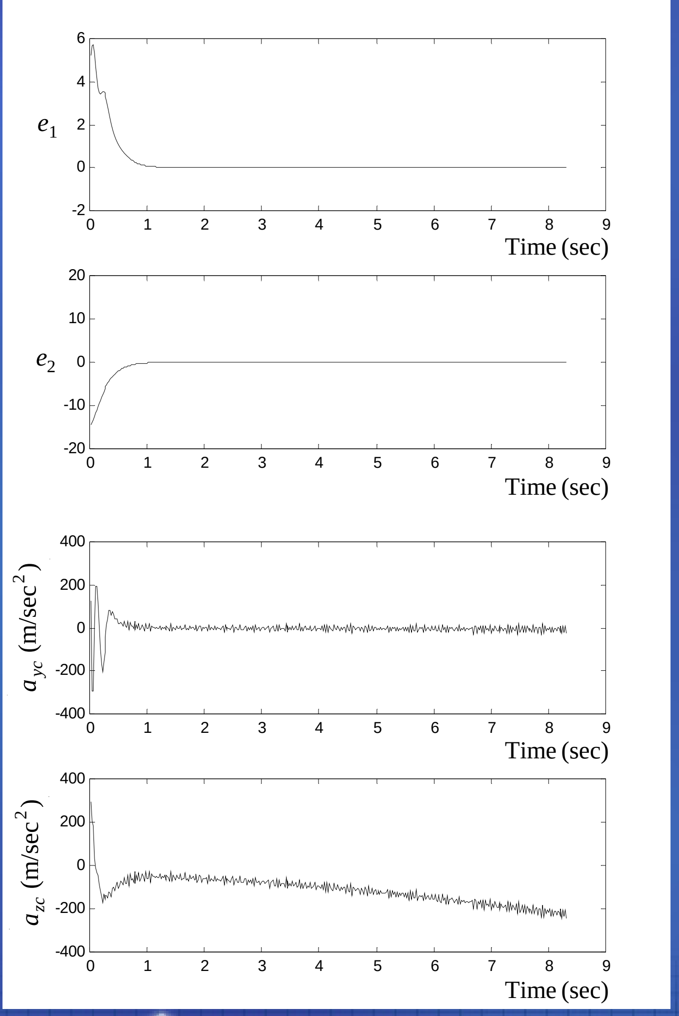
Engagement scenario 2 with feedback linearization guidance law.

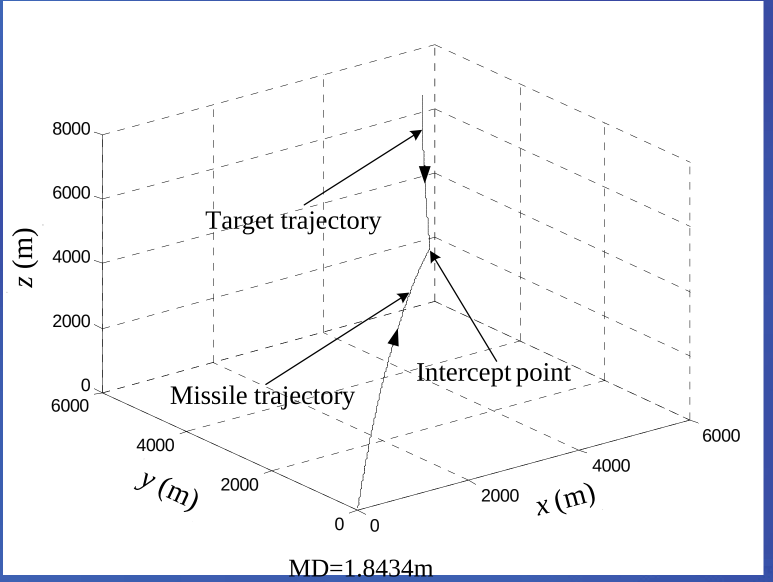




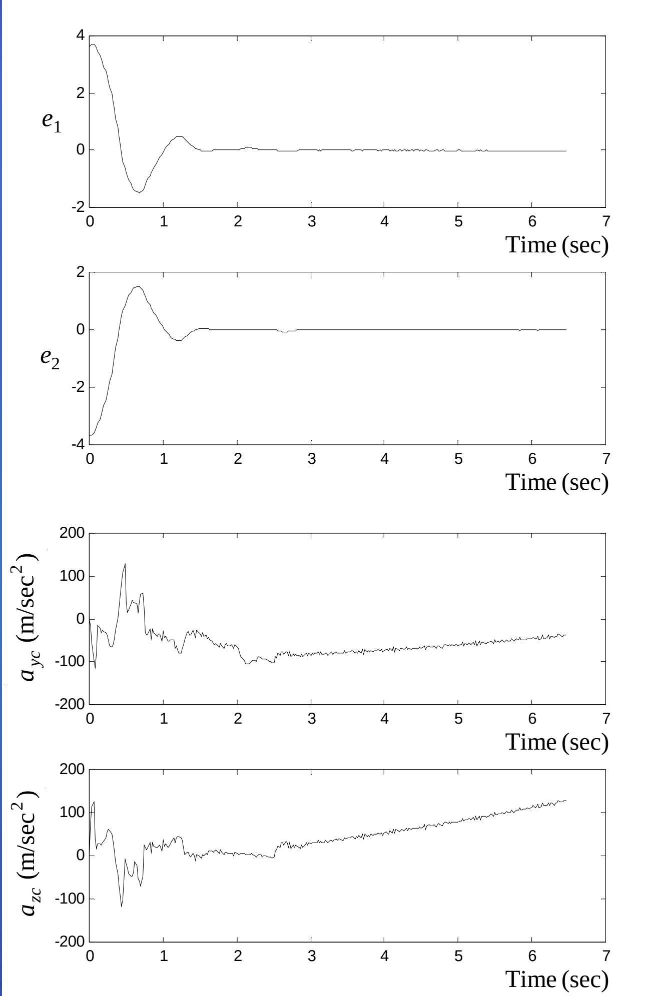
MD=3.758m

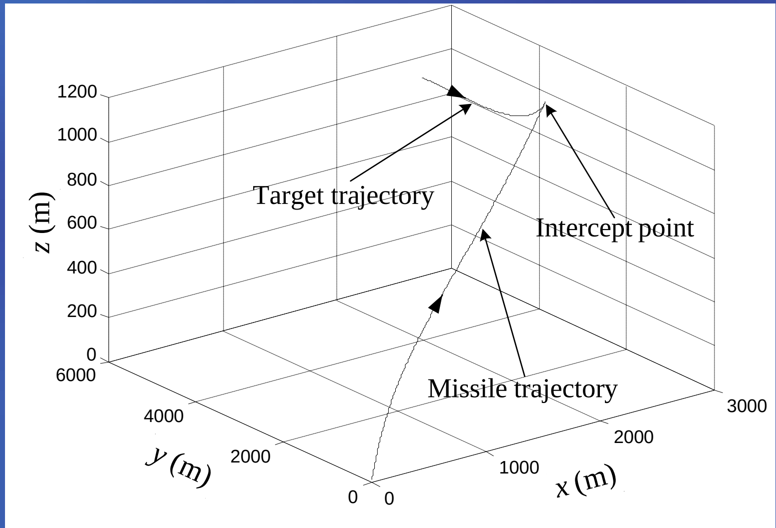
Engagement scenario 3 with feedback linearization guidance law.





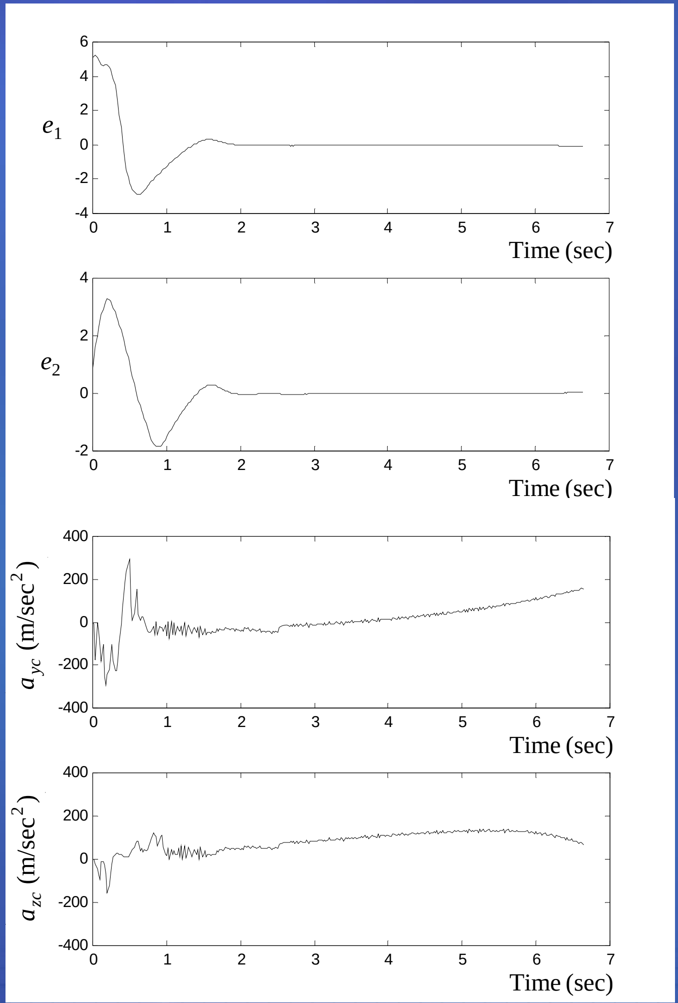
* Engagement scenario 1 with adaptive CMAC-based guidance law.

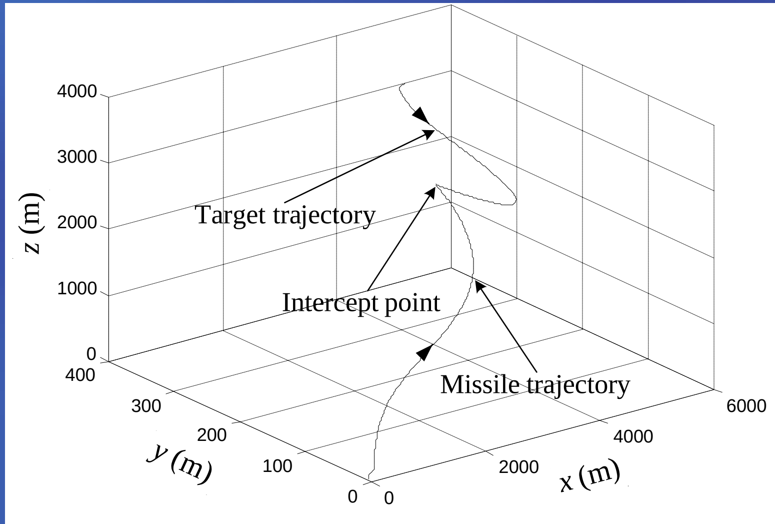




MD=0.5737m

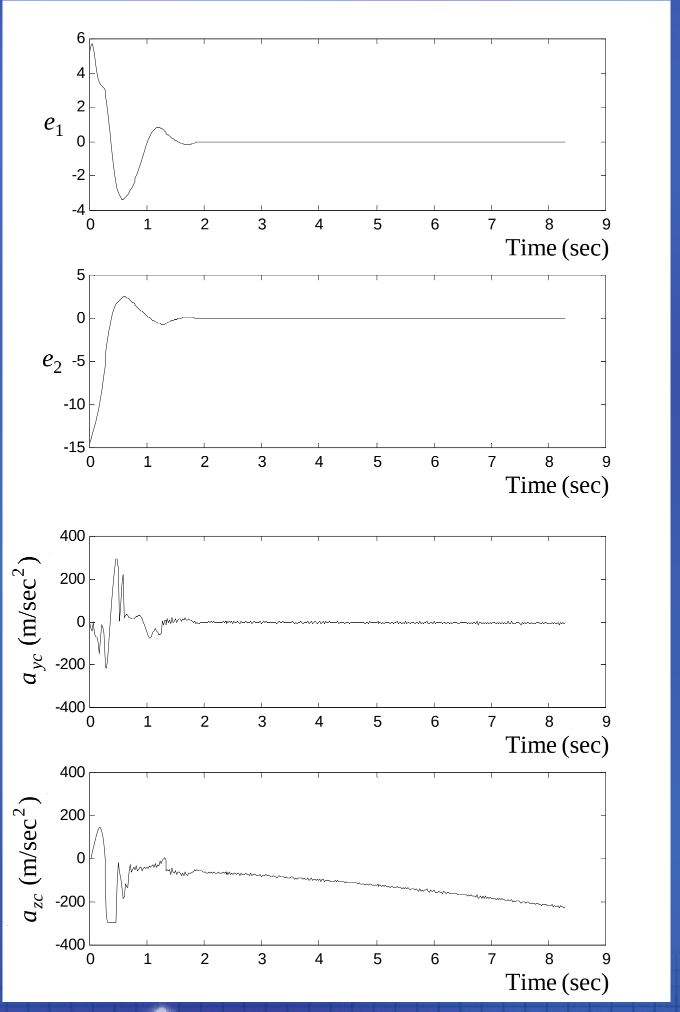
Engagement scenario 2 with adaptive CMAC-based guidance law.

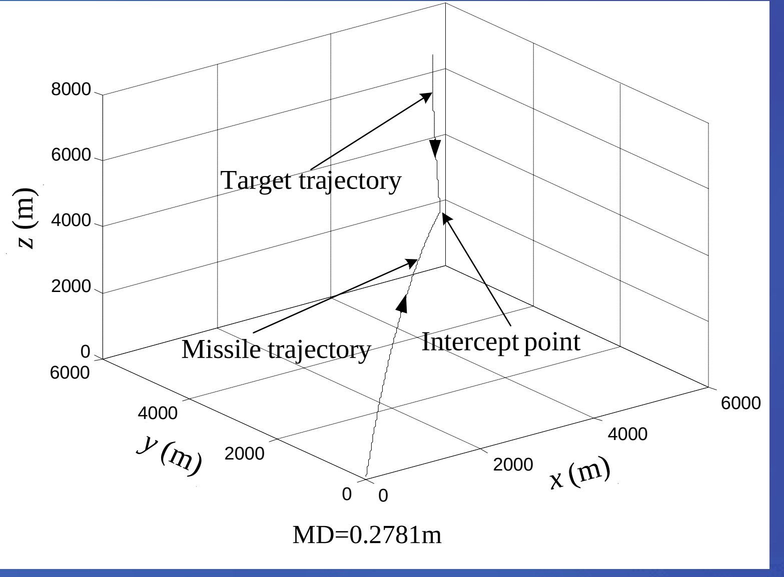




MD=1.5612m

Engagement scenario 3 with adaptive CMAC-based guidance law.





Comparison of Miss-distance

Scenario / Guidance Law	Scenario 1	Scenario 2	Scenario 3
Feedback Linearization Guidance Law	4.4539	3.758	1.8434
Adaptive CMAC-based Guidance Law	0.5737	1.5612	0.2781

Summary

- CMAC guidance law can achieve satisfactory performance for different engagement scenarios.
- CMAC guidance law performs better than the feedback linearization guidance law.

3. Linear Piezoelectric Ceramic Motor (LPCM) Control Using Adaptive CMAC

- Structure of LPCM

Unknown dynamic equation

$$\ddot{x}(t) = f(x;t) + g(x;t)u(t) + d(x;t) \tag{3.1}$$

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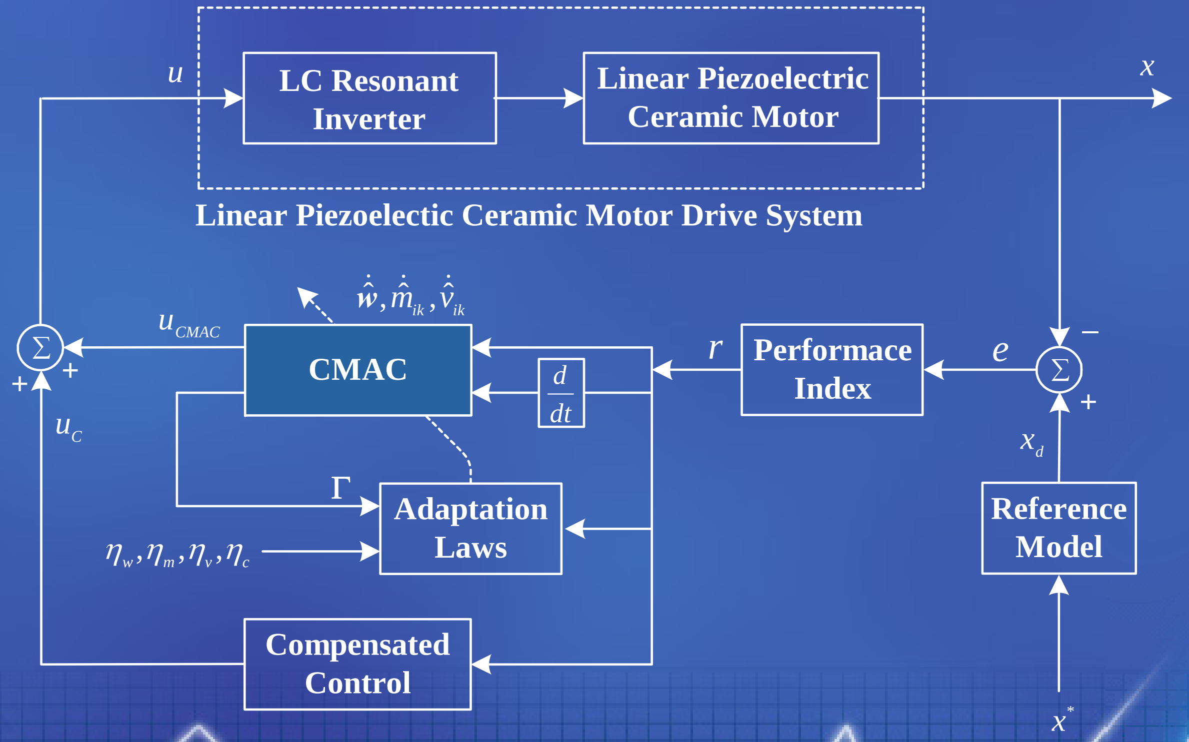
Adaptive CMAC Control System

The adaptive CMAC control law

$$u = u_{CMAC} + u_C \tag{3.2}$$



- Tracking error

$$e = x_d - x \tag{3.3}$$

- Performance index

$$r(t) = \dot{e} + k_2 e + k_1 \int_0^t e(\tau)d\tau \tag{3.4}$$

- Ideal control law

$$u_T = g(x;t)^{-1}[\ddot{x}_d - f(x;t) - d(x;t) + k_2\dot{e} + k_1 e] \tag{3.5}$$

Substituting (3.5) into (3.1), then

$$\dot{r}(t) = \ddot{e} + k_2\dot{e} + k_1 e = 0 \tag{3.6}$$

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- Adaptive CMAC control

$$u = u_{CMAC}(\boldsymbol{q}, \hat{\boldsymbol{w}}, \hat{\boldsymbol{m}}, \hat{\boldsymbol{v}}) + u_C = \hat{\boldsymbol{w}}^T \Gamma + u_C \qquad (3.7)$$

- Error equation

$$\dot{r}(t) = g(x;t)[u_T - u_{CMAC}(\boldsymbol{q}, \hat{\boldsymbol{w}}, \hat{\boldsymbol{m}}, \hat{\boldsymbol{v}}) - u_C] \qquad (3.8)$$

- A minimum approximation error

$$\varepsilon = u_T - u^*_{CMAC}(\boldsymbol{q}, \boldsymbol{w}^*, \boldsymbol{m}^*, \boldsymbol{v}^*) = u_T - \boldsymbol{w}^{*T}\Gamma \qquad (3.9)$$

According to (3.9), (3.8) can be rewritten as

$$\begin{aligned}\dot{r}(t) &= g(x;t)[\varepsilon + (\boldsymbol{w}^* - \hat{\boldsymbol{w}})^T \Gamma - u_C] \\ &= g(x;t)[\varepsilon + \tilde{\boldsymbol{w}}^T \Gamma - u_C] \qquad (3.10)\end{aligned}$$

Theorem 3.1: The adaptive CMAC control law is designed as (3.8), in which the adaptation law

$$\dot{\hat{w}} = \beta_w r(t) g(x;t) \Gamma \tag{3.11}$$

and the compensated control is designed as

$$u_C = \hat{\delta}\,\mathrm{sgn}[r(t)g(x;t)] \tag{3.12}$$

with bound estimation algorithm given in

$$\dot{\hat{\delta}} = \beta_c \,|\, r(t) g(x;t) \,| \tag{3.13}$$

then the stability of the control system can be guaranteed.

Proof: A Lyapunov function is chosen as

$$V(t) = \frac{1}{2} r^2(t) + \frac{1}{2\beta_w} \tilde{\boldsymbol{w}}^T \tilde{\boldsymbol{w}} + \frac{1}{2\beta_c} \tilde{\delta}^2 \quad (3.14)$$

$$\dot{V}(t) = r(t) g(x;t)[\varepsilon + \tilde{\boldsymbol{w}}^T \Gamma - u_C] - \frac{1}{\beta_w} \tilde{\boldsymbol{w}}^T \dot{\hat{\boldsymbol{w}}} - \frac{1}{\beta_c} \tilde{\delta} \dot{\hat{\delta}} \quad (3.15)$$

Substituting (3.11)-(3.13) into (3.15), gives

$$\begin{aligned} \dot{V}(t) &= r(t) g(x;t) \varepsilon - r(t) g(x;t) u_C - \frac{1}{\beta_c} \tilde{\delta} \tilde{\delta} \\ &\leq | r(t) g(x;t) \| \varepsilon | - r(t) g(x;t) u_C - \frac{1}{\beta_c} [\delta - \hat{\delta}] \dot{\hat{\delta}} \quad (3.16) \\ &= -(\delta - |\varepsilon|) | r(t) g(x;t) | \leq 0 \end{aligned}$$

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- According to the gradient descent method,

$$\dot{\hat{w}}_k = \beta_w r(t) g(x;t) b_k \equiv -\beta_w \frac{\partial V}{\partial \hat{w}_k} = -\beta_w \frac{\partial V}{\partial u_{CMAC}} b_k \qquad (3.17)$$

- The adaptation laws of means and variances

$$\dot{\hat{m}}_{ik} = -\beta_m \frac{\partial V}{\partial u_{CMAC}} \sum_{k=1}^{n_R} \frac{\partial u_{CMAC}}{\partial b_k} \frac{\partial b_k}{\partial \phi_{ik}} \frac{\partial \phi_{ik}}{\partial m_{ik}}$$

$$= \beta_m r(t) g(x;t) \sum_{k=1}^{n_R} \hat{w}_k b_k \frac{2(q_i - m_{ik})}{(v_{ik})^2} \qquad (3.18)$$

$$\dot{\hat{v}}_{ik} = -\beta_v \frac{\partial V}{\partial u_{CMAC}} \sum_{k=1}^{n_R} \frac{\partial u_{CMAC}}{\partial b_k} \frac{\partial b_k}{\partial \phi_{ik}} \frac{\partial \phi_{ik}}{\partial v_{ik}}$$

$$= \beta_v r(t) g(x;t) \sum_{k=1}^{n_R} \hat{w}_k b_k \frac{2(q_i - m_{ik})^2}{(v_{ik})^3} \qquad (3.19)$$

- The $g(x;t)$ in the tuning algorithms can be reorganized as $|g(x;t)|\,\mathrm{sgn}[g(x;t)]$ in practical applications.
- The adaptation laws can be reconstructed as

$$\dot{\hat{w}} = \eta_w r(t)\,\mathrm{sgn}[g(x;t)]\Gamma \tag{3.20}$$

$$u_C = \hat{\delta}\,\mathrm{sgn}[r(t)]\,\mathrm{sgn}[g(x;t)] \tag{3.21}$$

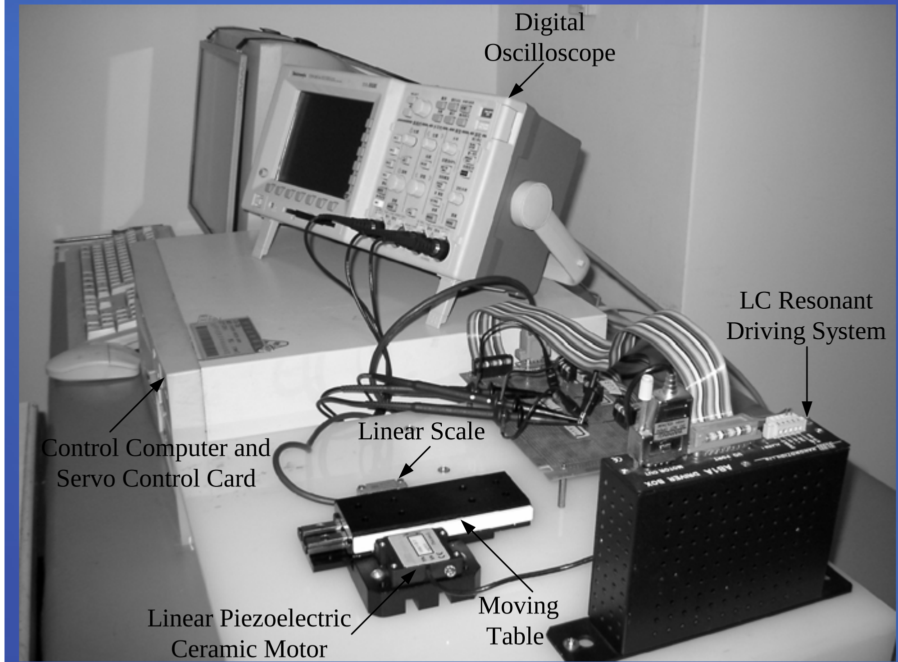
$$\dot{\hat{\delta}} = \eta_c |r(t)| \tag{3.22}$$

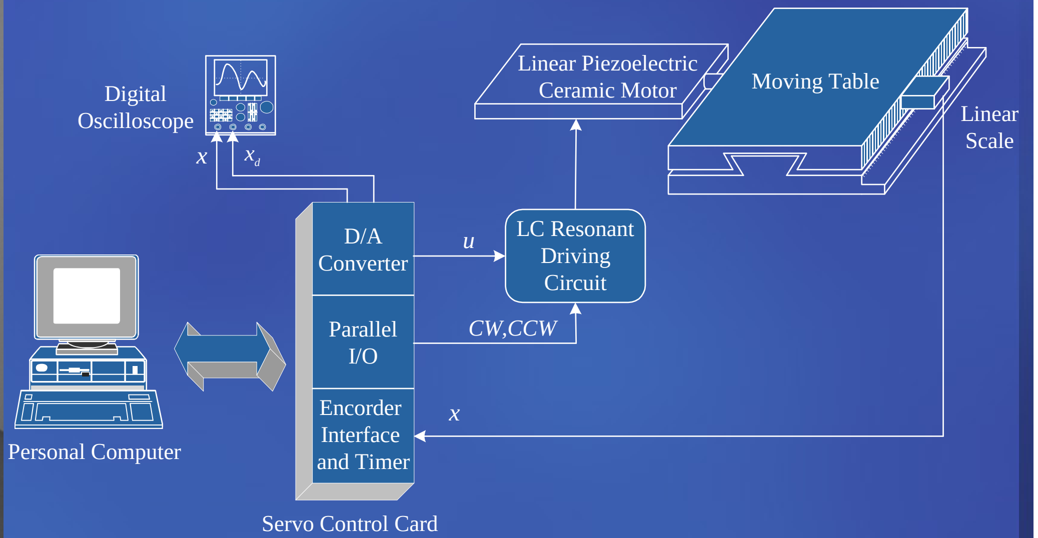
$$\dot{\hat{m}}_{ik} = \eta_m r(t)\,\mathrm{sgn}[g(x;t)]\sum_{k=1}^{n_R} \hat{w}_k b_k \frac{2(q_i - m_{ik})}{(v_{ik})^2} \tag{3.23}$$

$$\dot{\hat{v}}_{ik} = \eta_v r(t)\,\mathrm{sgn}[g(x;t)]\sum_{k=1}^{n_R} \hat{w}_k b_k \frac{2(q_i - m_{ik})^2}{(v_{ik})^3} \tag{3.24}$$

Experimental Results

- The PC-based experimental control system.

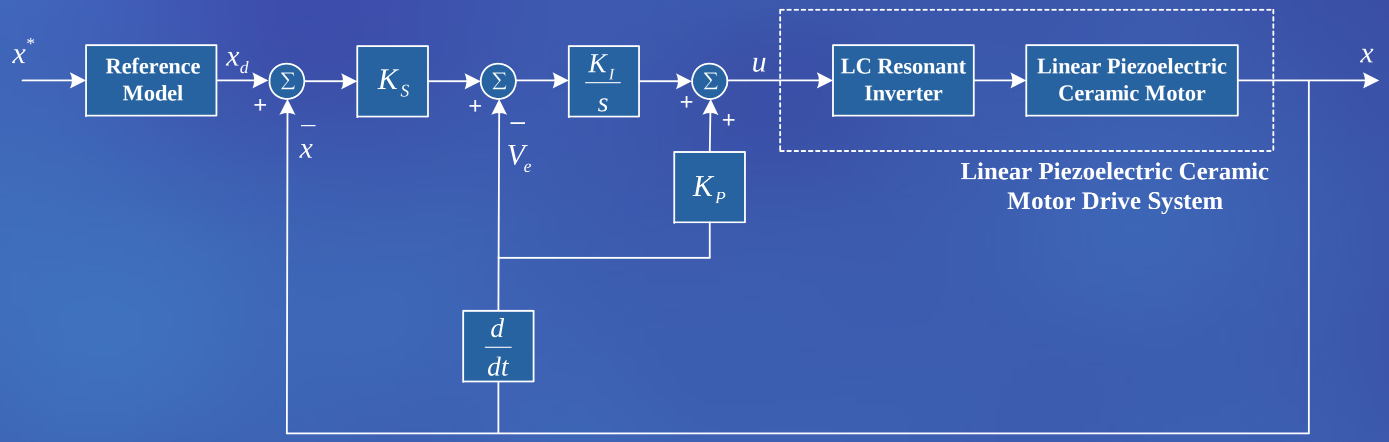




- The reference model for the periodic step command:

$$\frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}=\frac{73.36}{s^2+17.13s+73.36} \tag{3.25}$$

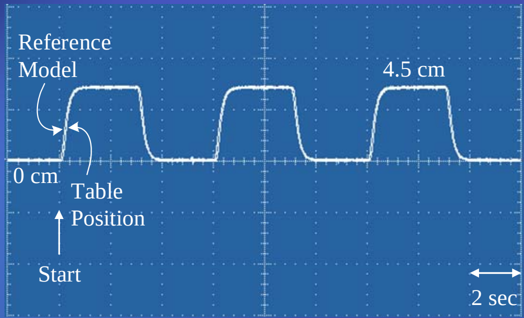
- Block diagram of PI position control system.



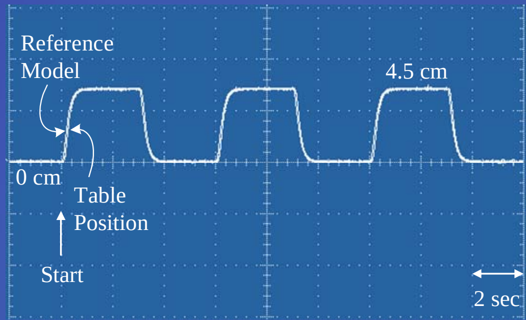
The parameters of the PI position control system are chosen as follows:

$$K_S = 20, \quad K_P = 1, \quad K_I = 25$$

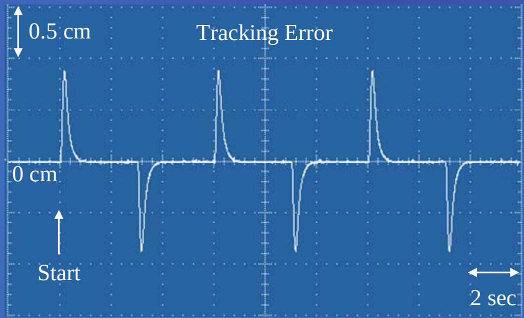
Experimental results of PI position control for LPCM due to periodic step command.



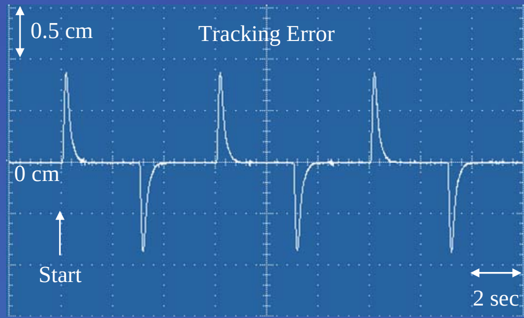
(a)



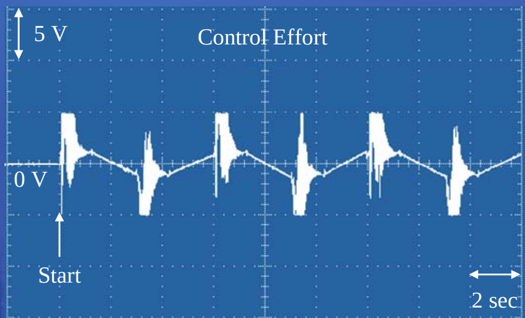
(d)



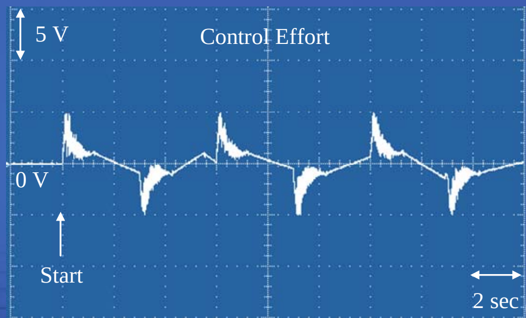
(b)



(e)



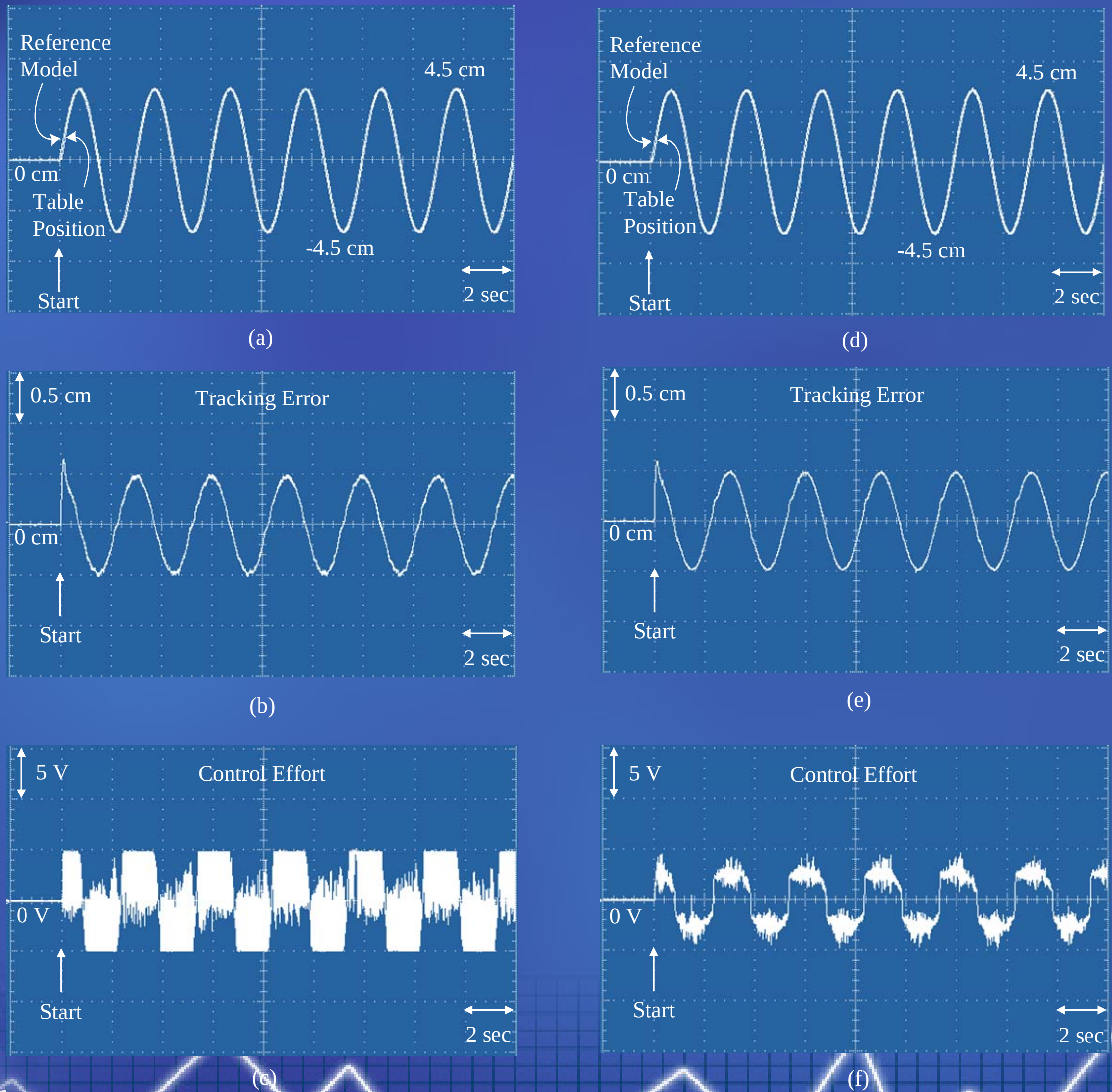
(c)



(f)

- No load case
- Load case

Experimental results of PI position control for LPCM due to sinusoidal command.



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- The adaptive CMAC control

Parameters

$$k_1 = 25,\ k_2 = 10,\ \eta_w = 0.04,\ \eta_m = \eta_v = 0.02,\ \eta_c = 0.01$$

$$\rho = 4,\ \ n_E = 5,\ \ n_B = 2\times 4,\ n_R = 2\times 2\times 4$$

The receptive-field basis functions are chosen as

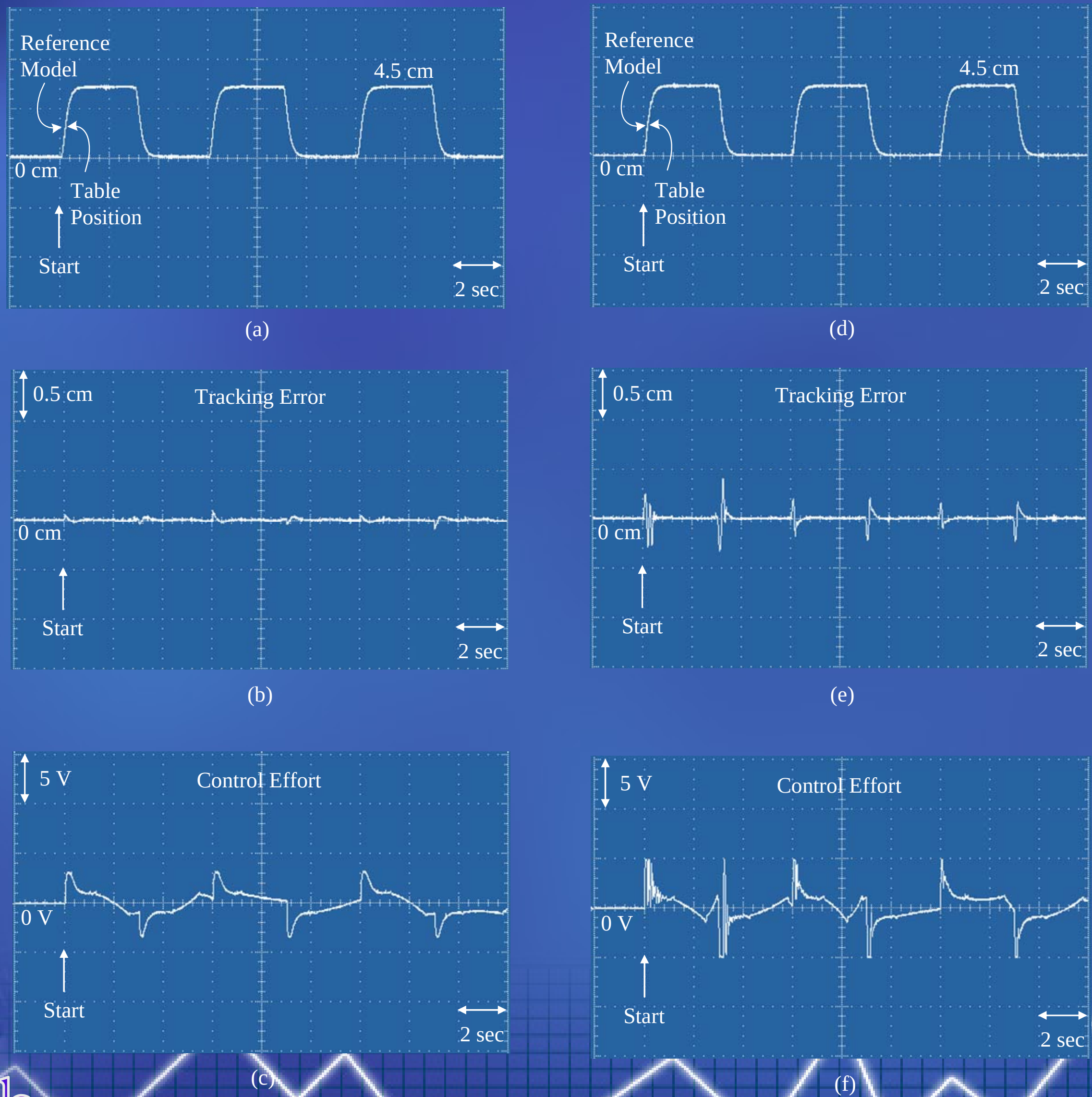
$$\phi_{ik}(q_i) = \exp[-(q_i - m_{ik})^2 / v_{ik}^2]$$

The initial values of the parameters are chosen as

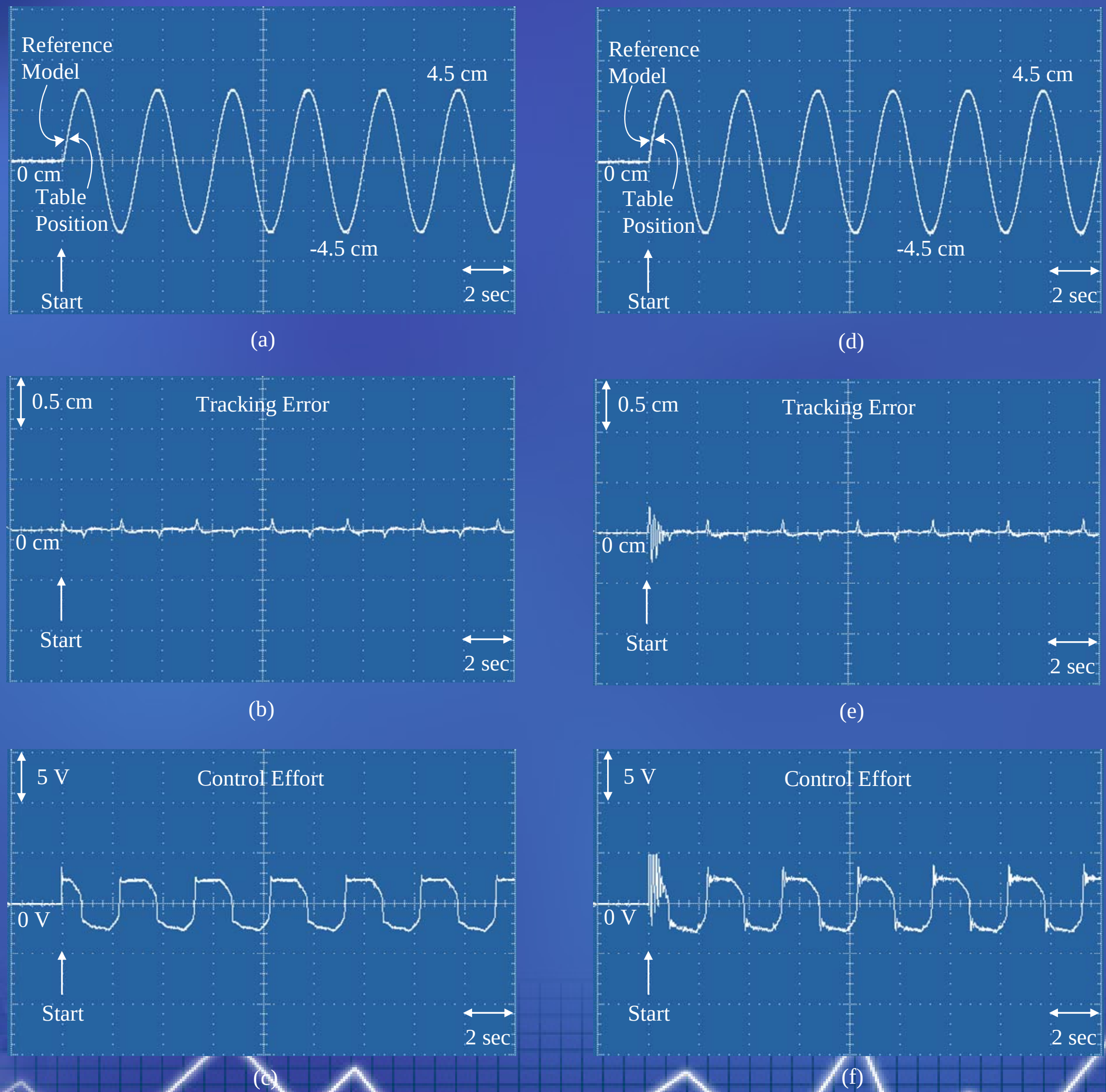
$$[m_{i1}, m_{i2},\ m_{i3}, m_{i4}, m_{i5}, m_{i6}, m_{i7}, m_{i8}]$$
$$= [-35, -25, -15, -5, 5, 15, 25, 35]$$

$$v_{ik} = 20$$

Experimental results of robust CMAC control for LPCM due to periodic step command.



Experimental results of robust CMAC control for LPCM due to sinusoidal command.



Summary

- The successful development of adaptive CMAC control system.
- The successful application of adaptive CMAC control for an LPCM.

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4. Recurrent CMAC Control for Unknown Uncertain Nonlinear Systems

- Problem Formulation
  - The nth-order nonlinear dynamic system

$$\begin{cases} x^{(n)} = f(\mathbf{x}) + g(\mathbf{x})u(t) + d(t) \\ y = x \end{cases} \tag{4.1}$$

  - The tracking error vector is defined as

$$\boldsymbol{E} \triangleq [e, \dot{e}, \cdots, e^{(n-1)}]^T \tag{4.2}$$

  - The ideal control law

$$u_I = \frac{1}{g(\boldsymbol{x})}[-f(\boldsymbol{x}) - d(t) + x_d^{(n)} + \boldsymbol{K}^T \boldsymbol{E}] \tag{4.3}$$

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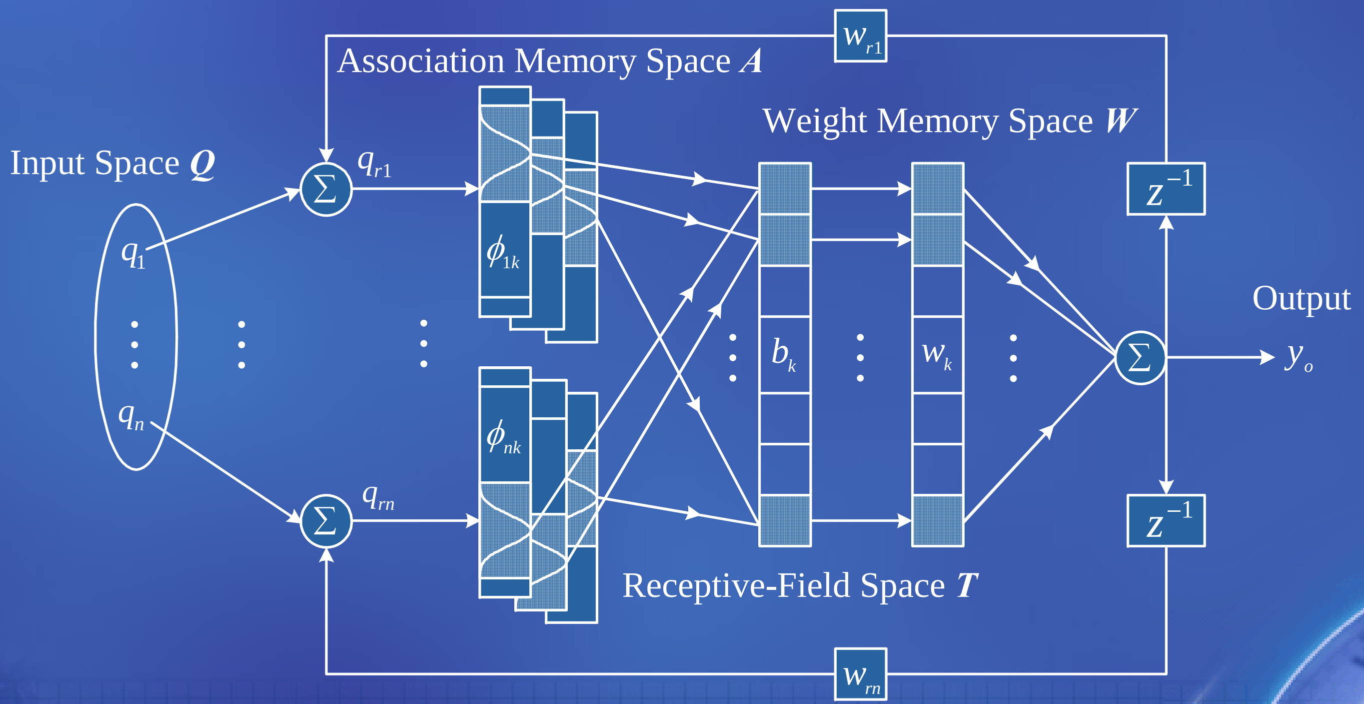
- The error dynamics

$$e^{(n)} + k_1 e^{(n-1)} + \cdots + k_n e = 0 \qquad (4.4)$$

Recurrent CMAC

- Architecture of a recurrent CMAC



The inputs of every block $\boldsymbol{q}_r = [q_{r1}, q_{r2}, \cdots, q_{rn}]^T \in \Re^n$ are represented as

$$\boldsymbol{q}_r = \boldsymbol{q} + \boldsymbol{w}_r \; y_o(N-1) \tag{4.5}$$

- The receptive-field basis function

$$\phi_{ik}(q_{ri}) = \exp\left[\frac{-(q_{ri} - m_{ik})^2}{v_{ik}^2}\right] \tag{4.6}$$

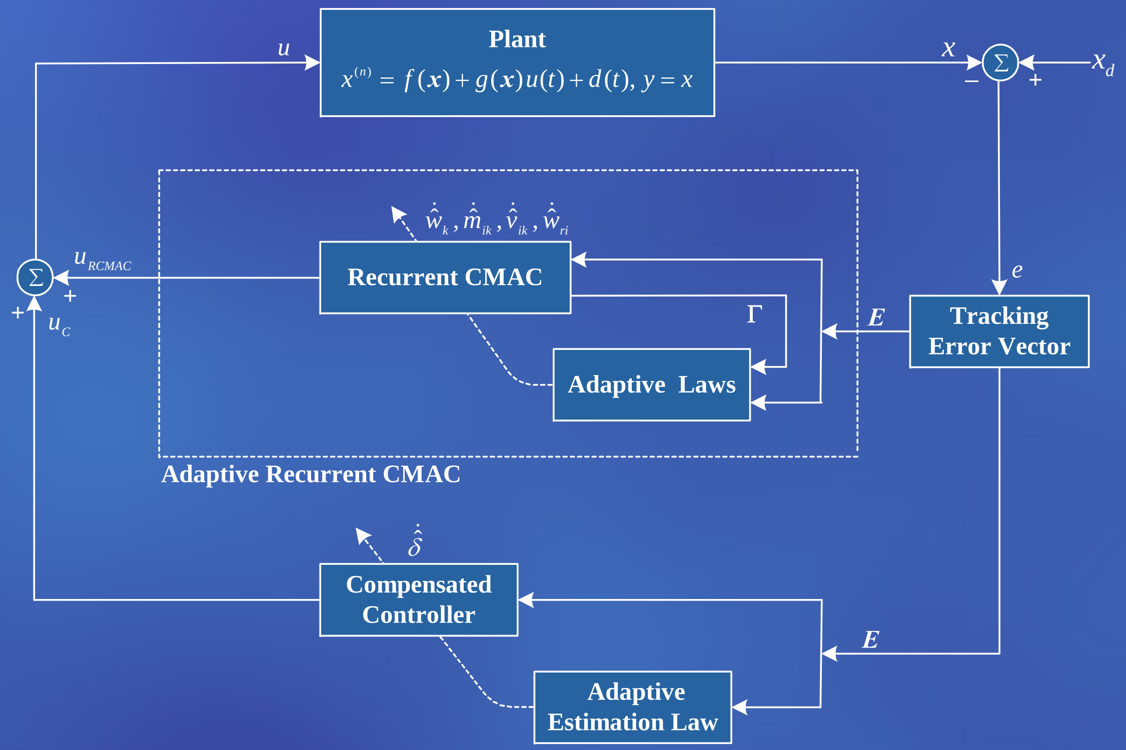
- The RCMAC is utilized to estimate the perfect control law, so that

$$u_{RCMAC}(\boldsymbol{q}_r, \boldsymbol{w}, \boldsymbol{m}, \boldsymbol{v}, \boldsymbol{w}_r) = y_o \equiv \boldsymbol{w}^T \Gamma \tag{4.7}$$

Recurrent CMAC Control System

Control law

$$u = u_{RCMAC} + u_C \qquad (4.8)$$



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- ***Theorem 4.1:*** The adaptive law of the recurrent CMAC is designed as

$$\dot{\hat{w}} = \beta_w \boldsymbol{E}^T \boldsymbol{P}\boldsymbol{B}_m \Gamma \qquad (4.9)$$

and the compensated controller is designed as

$$u_C = \hat{\delta}\,\mathrm{sgn}(\boldsymbol{E}^T \boldsymbol{P}\boldsymbol{B}_m) \qquad (4.10)$$

with the adaptive estimation law given in

$$\dot{\hat{\delta}} = \beta_c \,|\, \boldsymbol{E}^T \boldsymbol{P}\boldsymbol{B}_m \,| \qquad (4.11)$$

where β_w and β_c are positive constants, then the stability of the control system can be guaranteed.

On-line parameter training algorithm

$$\dot{\hat{w}}_k = \beta_w \boldsymbol{E}^T \boldsymbol{P}\boldsymbol{B}_m b_k(\boldsymbol{q}_r, \boldsymbol{m}_k, \boldsymbol{v}_k, \boldsymbol{w}_r) \equiv -\beta_w \frac{\partial V}{\partial u_{RCMAC}} \frac{\partial u_{RCMAC}}{\partial \hat{w}_k}$$

$$= -\beta_w \frac{\partial V}{\partial u_{RCMAC}} b_k(\boldsymbol{q}_r, \boldsymbol{m}_k, \boldsymbol{v}_k, \boldsymbol{w}_r) \qquad (4.11)$$

- The adaptive laws of means, variances and recurrent weights:

$$\dot{\hat{m}}_{ik} = \beta_m \boldsymbol{E}^T \boldsymbol{P}\boldsymbol{B}_m \hat{w}_k\, b_k \frac{2(q_{ri} - m_{ik})}{v_{ik}^2} \tag{4.12}$$

$$\dot{\hat{v}}_{ik} = \beta_v \boldsymbol{E}^T \boldsymbol{P}\boldsymbol{B}_m \hat{w}_k\, b_k \frac{2(q_{ri} - m_{ik})^2}{v_{ik}^3} \tag{4.13}$$

$$\begin{aligned}\dot{\hat{w}}_{ri} &= -\beta_r \frac{\partial V}{\partial u_{RCMAC}} \frac{\partial u_{RCMAC}}{\partial b_k} \frac{\partial b_k}{\partial \phi_{ik}} \frac{\partial \phi_{ik}}{\partial q_{ri}} \frac{\partial q_{ri}}{\partial \hat{w}_{ri}} \\ &= \beta_r \boldsymbol{E}^T \boldsymbol{P}\boldsymbol{B}_m \hat{w}_k\, b_k \frac{2(m_{ik} - q_{ri})}{v_{ik}^2} u_{RCMAC}(N-1)\end{aligned} \tag{4.14}$$

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- If $g(\mathbf{x})$ is unknown:

$$\dot{\hat{\boldsymbol{w}}} = \eta_w \boldsymbol{E}^T \boldsymbol{P}\boldsymbol{B}_r \Gamma \tag{4.15}$$

$$u_C = \hat{\delta}\,\mathrm{sgn}(\boldsymbol{E}^T \boldsymbol{P}\boldsymbol{B}_r) \tag{4.16}$$

$$\dot{\hat{\delta}} = \eta_c \,|\, \boldsymbol{E}^T \boldsymbol{P}\boldsymbol{B}_r \,| \tag{4.17}$$

$$\dot{\hat{m}}_{ik} = \eta_m \boldsymbol{E}^T \boldsymbol{P}\boldsymbol{B}_r \hat{w}_k\, b_k \frac{2(q_{ri} - m_{ik})}{v_{ik}^2} \tag{4.18}$$

$$\dot{\hat{v}}_{ik} = \eta_v \boldsymbol{E}^T \boldsymbol{P}\boldsymbol{B}_r \hat{w}_k\, b_k \frac{2(q_{ri} - m_{ik})^2}{v_{ik}^3} \tag{4.19}$$

$$\dot{\hat{w}}_{ri} = \eta_r \boldsymbol{E}^T \boldsymbol{P}\boldsymbol{B}_r \hat{w}_k\, b_k \frac{2(m_{ik} - q_{ri})}{v_{ik}^2} u_{RCMAC}(N-1) \tag{4.20}$$

where $\eta_j = \beta_j |\, g(x;t) \,|$ and $\mathbf{B}_r = [0, 0, \cdots, 1]^T \in \Re^n$

Illustrative Examples

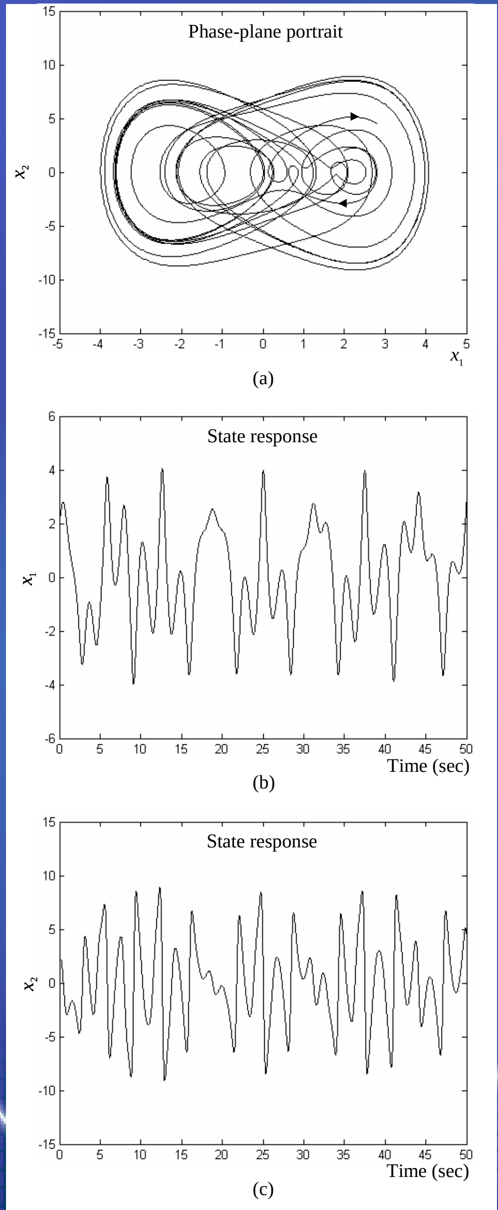
- *Example 4.1:* Consider the Duffing forced oscillation system

$$\ddot{x} + 0.1\dot{x} + x^3 = u + 12\cos(t) \qquad (4.21)$$

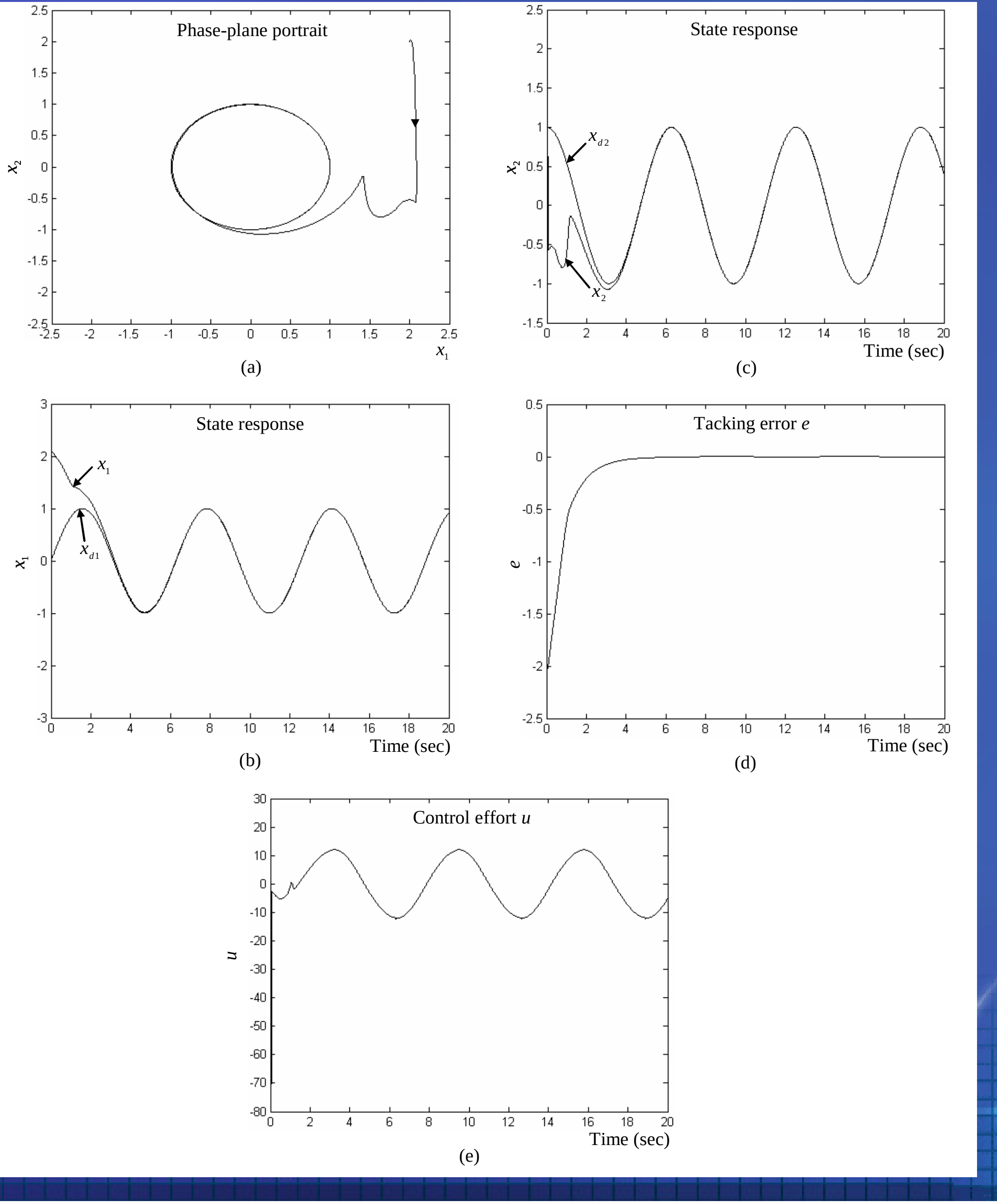
It can be rewritten as

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} (f + gu + d) \qquad (4.22)$$

Simulated results of the Duffing forced oscillation system (without control)



Simulated results of RCMAC control for the Duffing forced oscillation system.



Example 4.2: The delta wing of the wing rock motion control

$$\ddot{q} = c_0 + c_1 q + c_2 \dot{q} + c_3 |q| \dot{q} + c_4 |\dot{q}| \dot{q} + c_5 q^3 + u \qquad (4.23)$$

The state equation

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} (f + gu + d) \qquad (4.24)$$

The aerodynamic parameters of the delta wing for a 25-deg angle of attack are chosen as

$$c_0 = 0,\ \ c_1 = -0.01859521,\ \ c_2 = 0.015162375,$$
$$c_3 = -0.06245153,\ \ c_4 = 0.00954708,$$
$$c_5 = 0.02145291$$

Two initial conditions:

a small initial condition

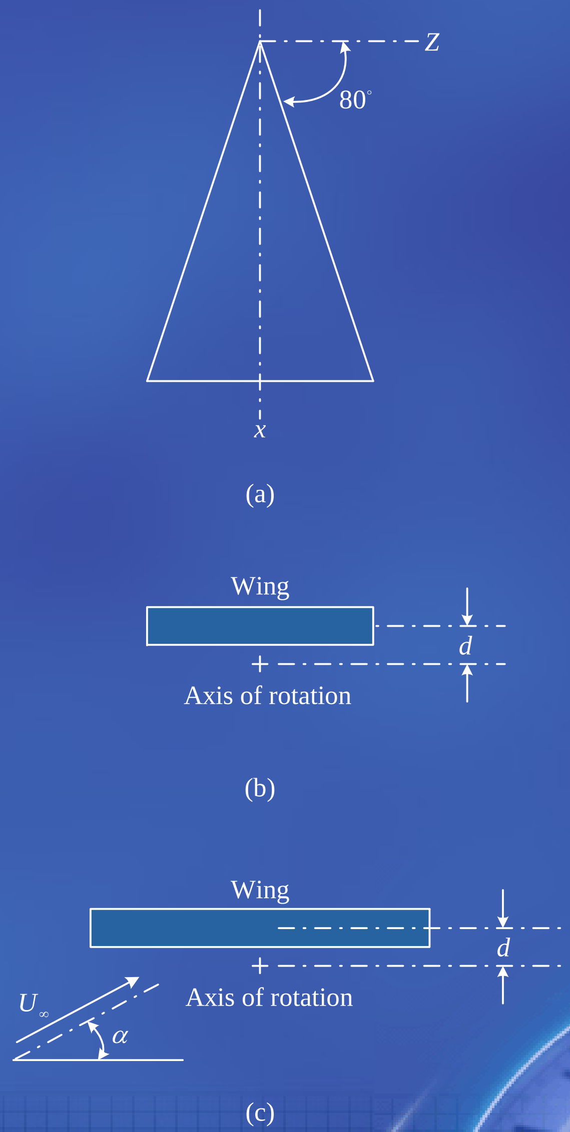
$$x_1(0) = 6\,\mathrm{deg}$$

$$x_2(0) = 3\,\mathrm{deg/sec}$$

a large initial condition

$$x_1(0) = 30\,\mathrm{deg}$$

$$x_2(0) = 10\,\mathrm{deg/sec}$$



The reference model is defined as

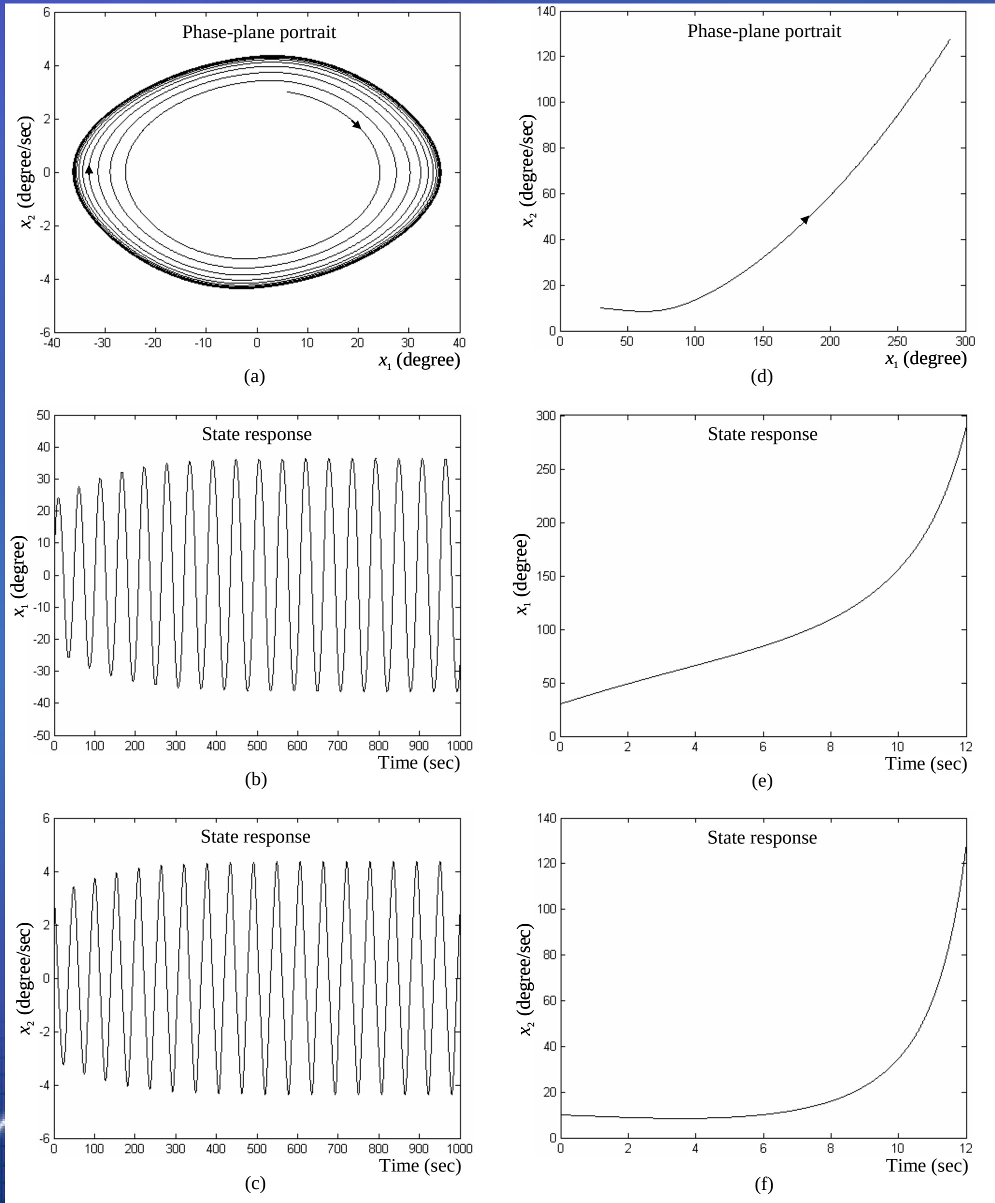
$$\begin{bmatrix} \dot{x}_{d1} \\ \dot{x}_{d2} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -0.64 & -1.6 \end{bmatrix} \begin{bmatrix} x_{d1} \\ x_{d2} \end{bmatrix} \qquad (4.25)$$

The parameters are selected as

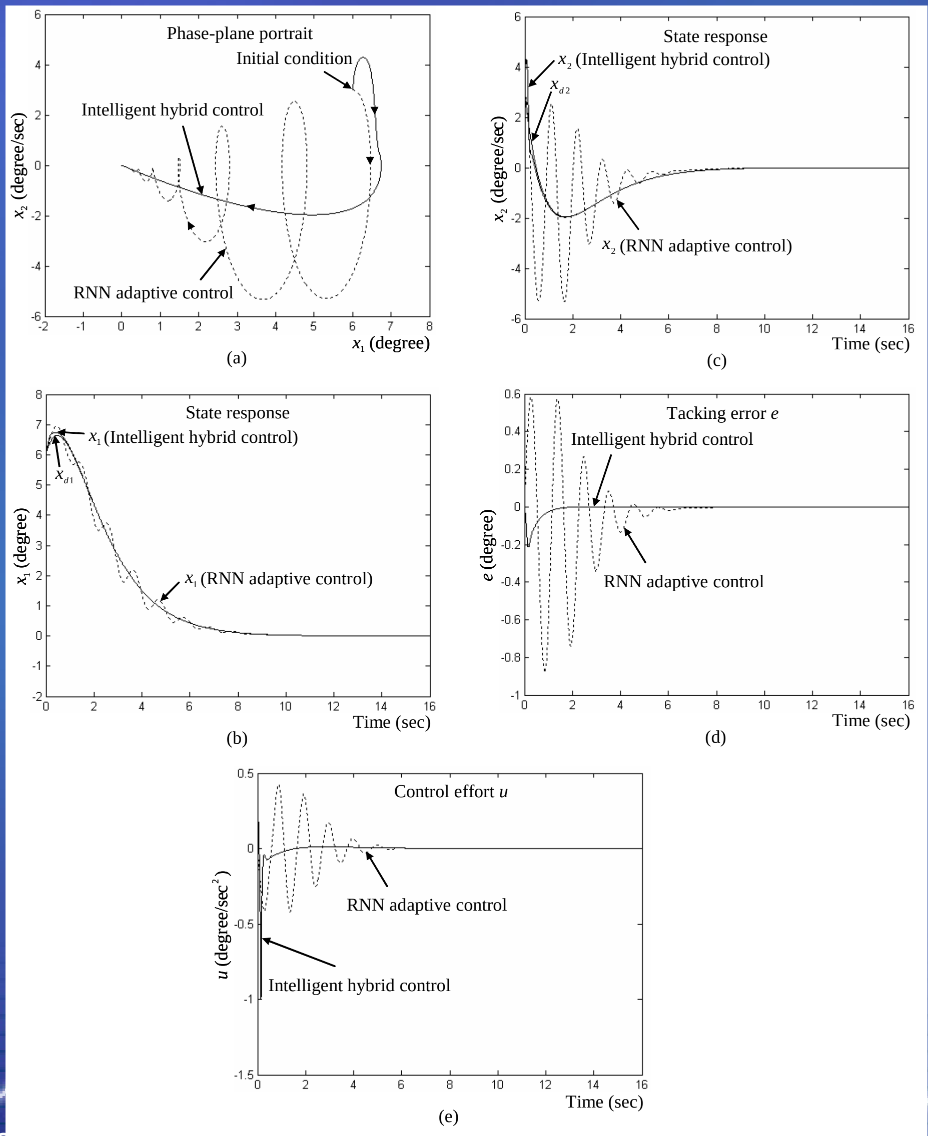
$$\boldsymbol{K} = [9, 6]^T, \quad \boldsymbol{R} = \begin{bmatrix} 90 & 9 \\ 9 & 2 \end{bmatrix}, \quad \boldsymbol{P} = \begin{bmatrix} 30 & 5 \\ 5 & 1 \end{bmatrix},$$

$$\eta_w = 6.25, \quad \eta_m = \eta_v = 0.75, \quad \eta_r = 0.01, \quad \eta_c = 0.05$$

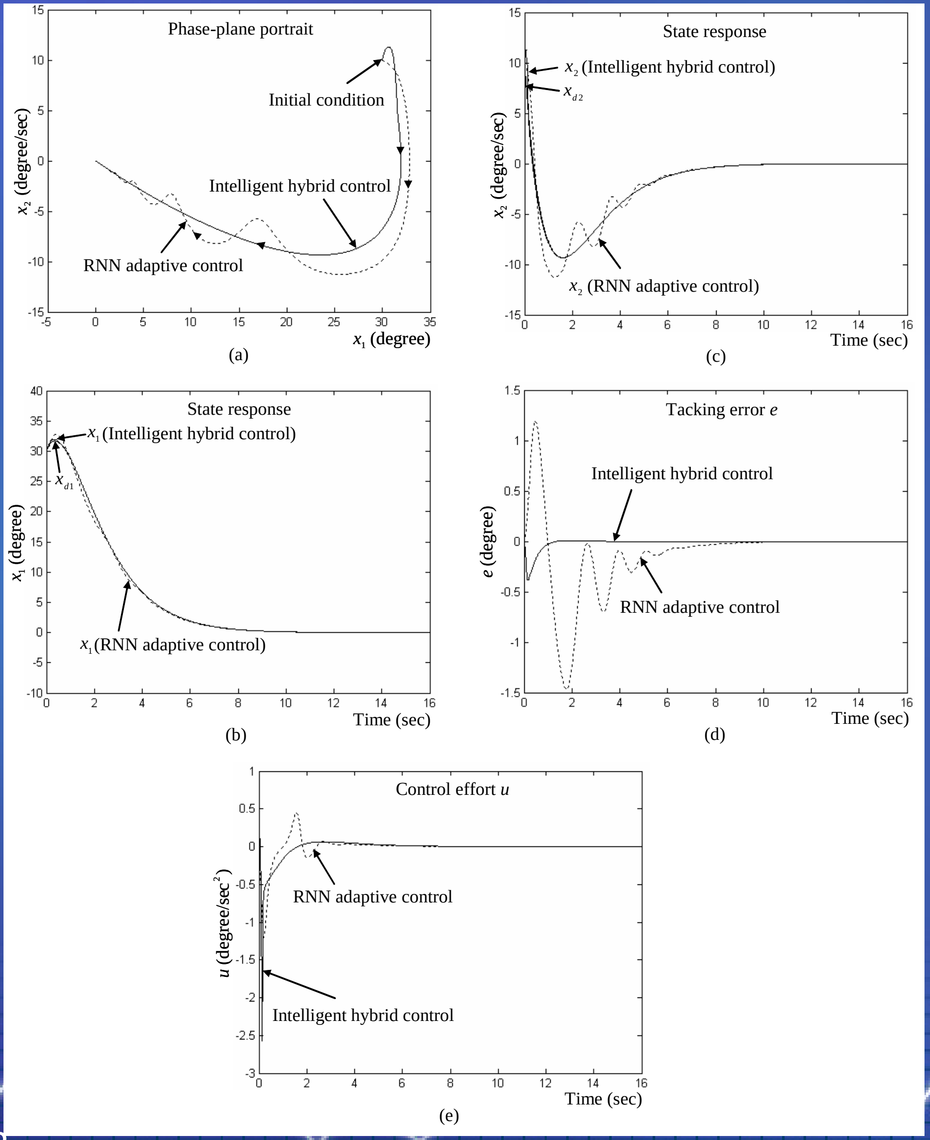
Simulated results of the wing rock motion system (without control)



Simulated results of RCMAC control and RNN control for small initial condition.



* Simulated results of RCMAC control and RNN control for large initial condition.



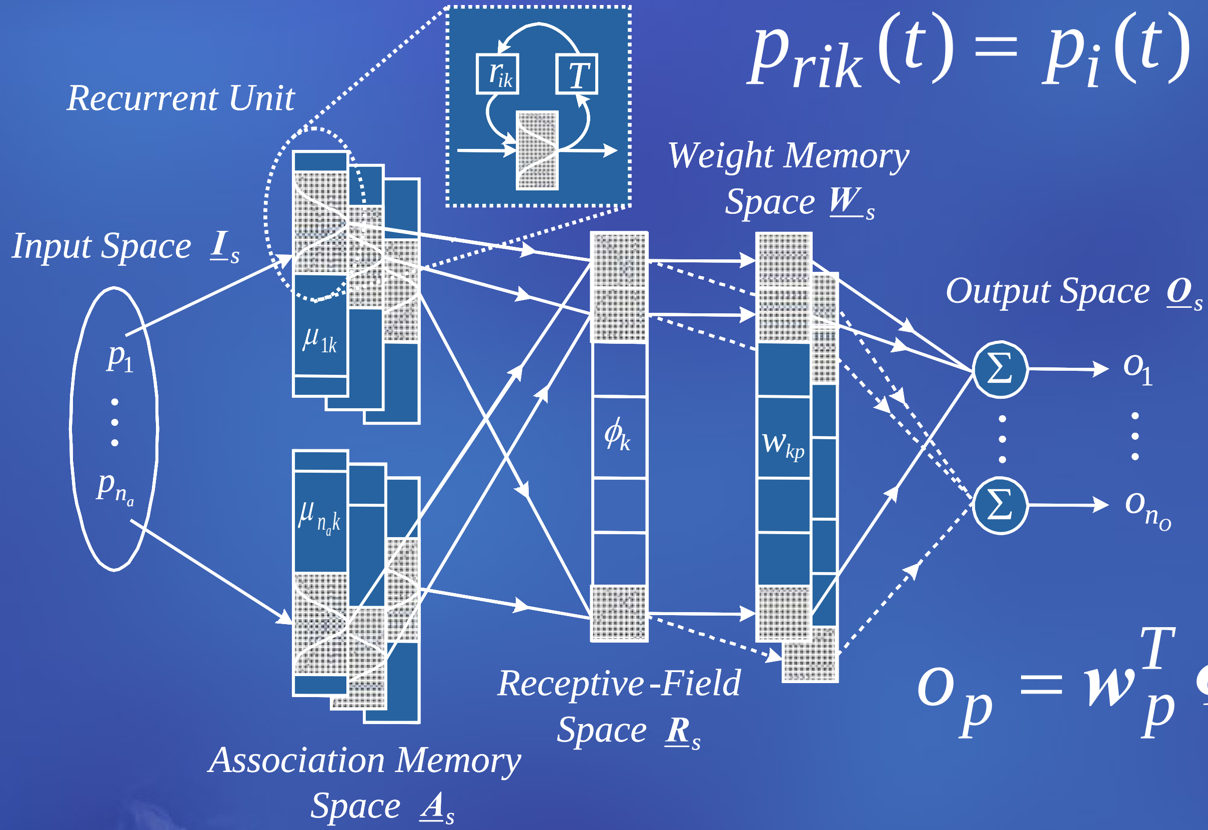
Summary

- An RCMAC control scheme has been proposed for a class of nonlinear dynamical system.
- RCMAC is introduced which has both the merits of RNN and conventional CMAC.

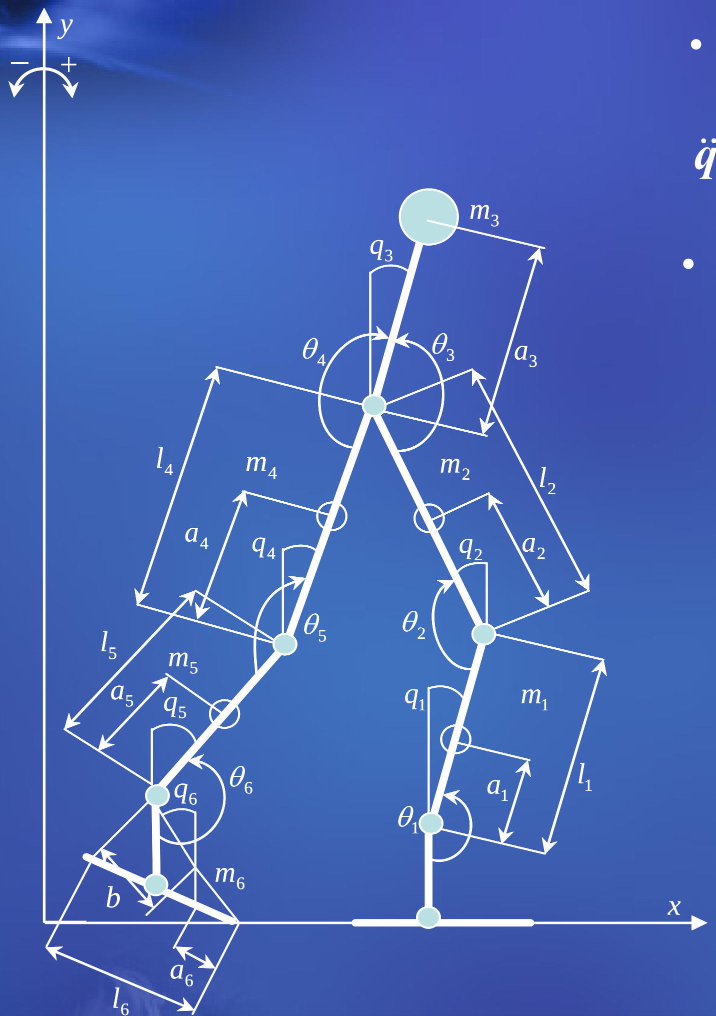
6. RCMAC Fault Accommodation Control of a Biped Robot

Structures of MIMO RCMAC

$$p_{rik}(t) = p_i(t) + r_{ik}\ \mu_{ik}(t-T) \quad (5.1)$$



$$o_p = \boldsymbol{w}_p^T \boldsymbol{\Phi} = \sum_{k=1}^{n_d} w_{kp}\,\phi_k \quad (5.2)$$



- A biped robot is subjected to nonlinear faults

$$\ddot{q} = M^{-1}(q)\left(\tau - C(q,\dot{q})\dot{q} - g(q) + \lambda(t-t_0)\bar{f}_t(q,\dot{q})\right) \quad (5.3)$$

- The unknown fault-occurrence time

$$\lambda(t-t_0) = \begin{cases} 0, & if \quad t < t_0 \\ 1, & if \quad t \geq t_0 \end{cases} \quad (5.4)$$

- In the absence of a fault

$$\ddot{q} = M^{-1}(q)[\tau - C(q,\dot{q})\dot{q} - g(q)] \qquad (5.5)$$

- Computed torque controller

$$\tau_0 = M(q)\left(\ddot{q}_d + 2K\dot{e} + K^2 e\right) + C(q,\dot{q})\dot{q} + g(q) \qquad (5.6)$$

where $e = q_d - q$ the tracking error vector

- Error dynamics $\ddot{e} + 2K\dot{e} + K^2 e = 0$ (5.7)

Fault occurs:

- Robust fault-accommodation controller

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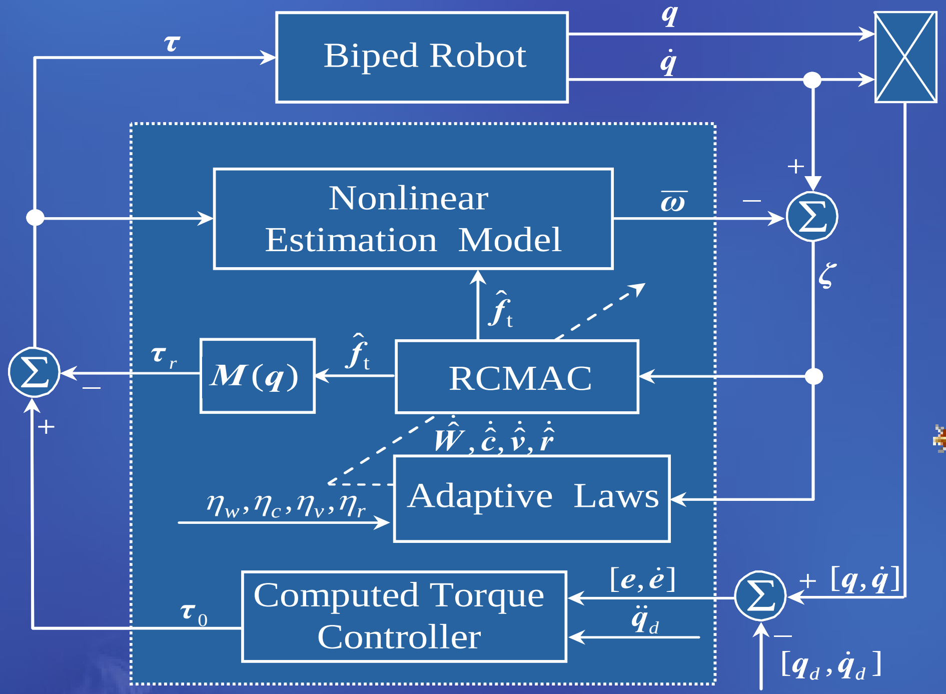
RCMAC-based fault accommodation control

$$\dot{\overline{\omega}} = \alpha_c(\dot{q} - \overline{\omega}) + M^{-1}(q)[\tau - C(q,\dot{q})\dot{q} - g(q)] + \hat{f}_t \qquad (5.8)$$



RCMAC-Based Fault-Tolerant Control System

- RCMAC

  To estimate the nonlinear fault

- Accommodation controller

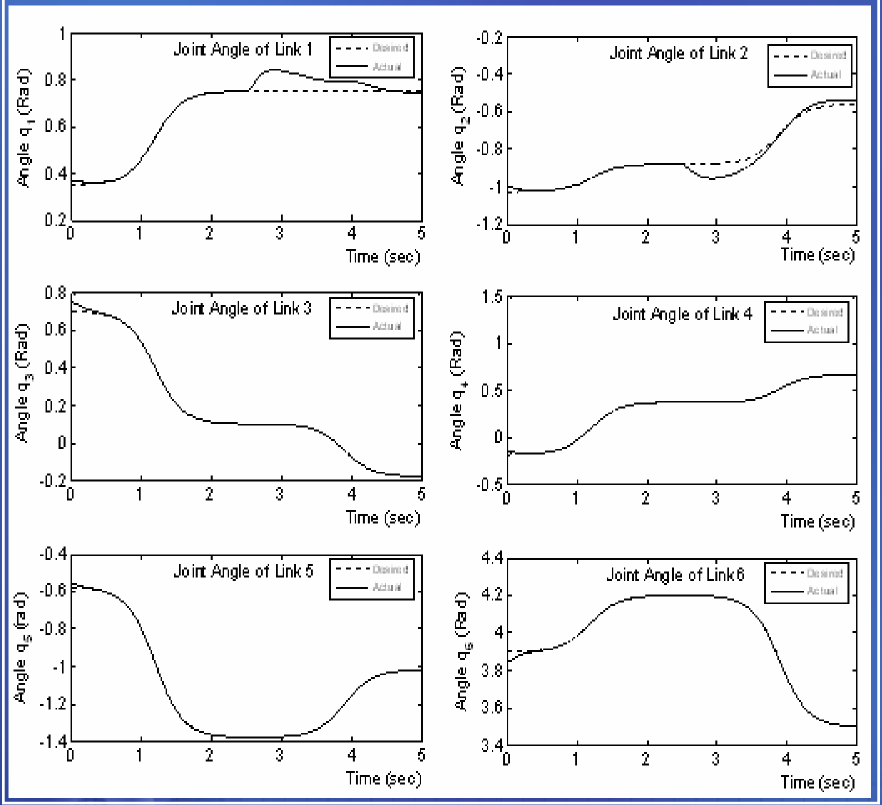
$$\tau_r = M(q)\hat{f}_t(q,\dot{q}) \qquad (5.9)$$

- Fault-accommodation control law

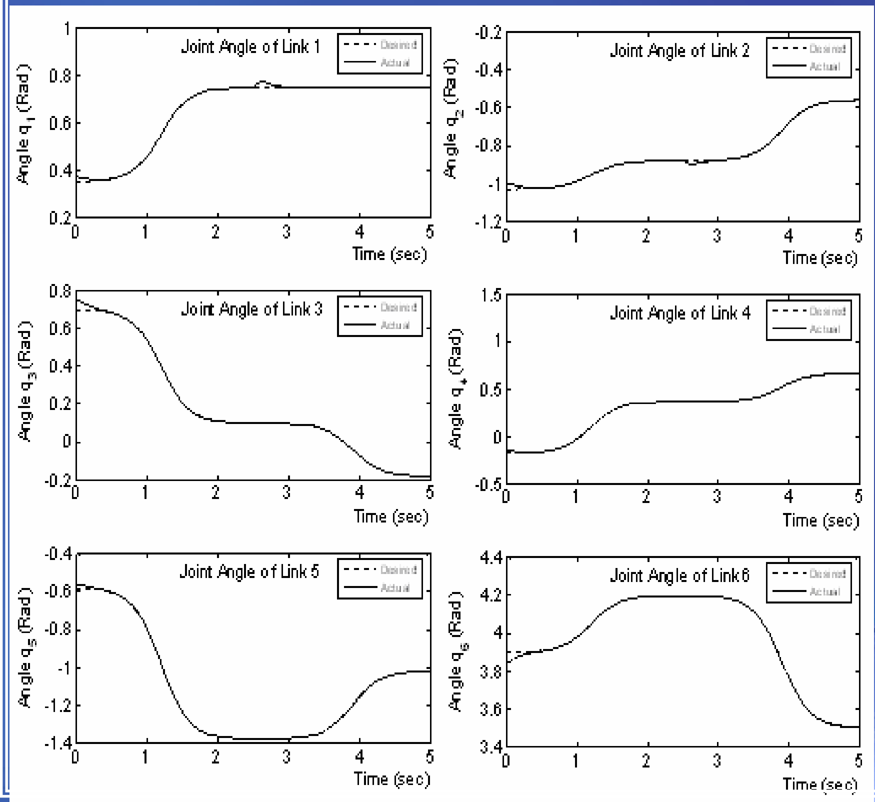
$$\tau = \tau_0 - \tau_r \qquad (5.10)$$

Simulation Results The joint angle of each link

CMAC

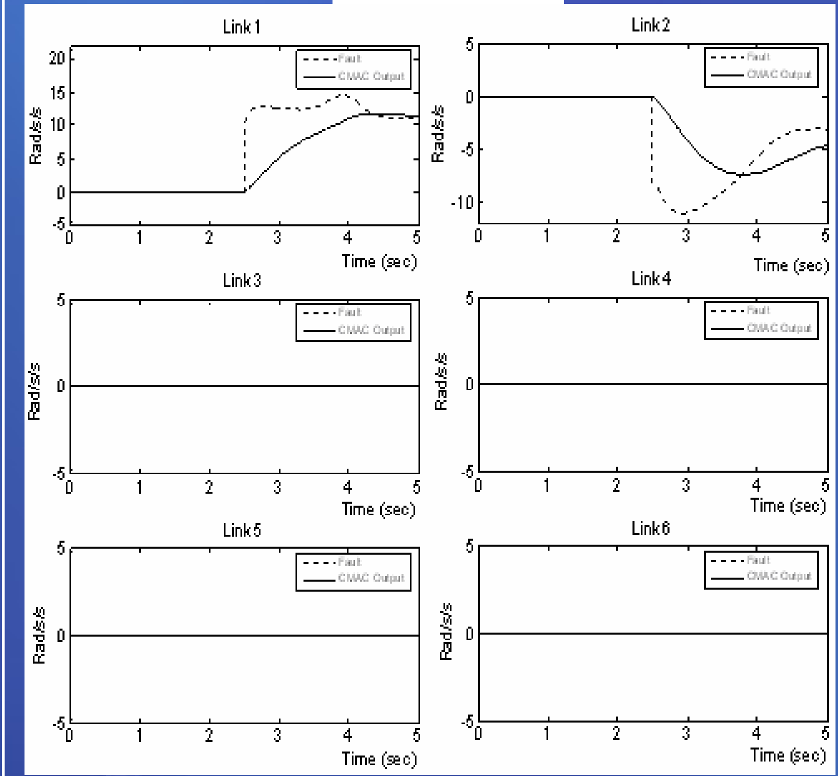


RCMAC

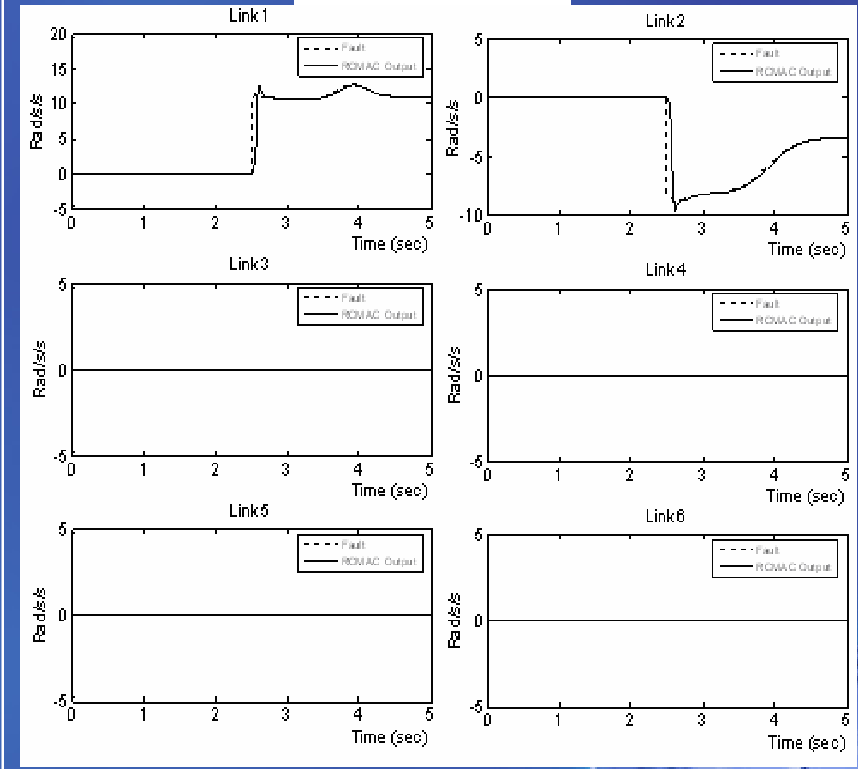


Simulation Results The fault function and the output

CMAC



RCMAC



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Summary

RCMAC can achieve favorable accommodation control for the faults of a biped robot.

Referred Papers

- Chih-Min Lin and Ya-Fu Peng, "Adaptive CMAC-based supervisory control for uncertain nonlinear systems," *IEEE Trans. System, Man, and Cybernetics Part B*, Vol. 34, No. 2, pp. 1248-1260, 2004.
- Chih-Min Lin, Ya-Fu Peng, and Chun-Fei Hsu, "Robust cerebellar model articulation controller design for unknown nonlinear systems," *IEEE Transactions on Circuits and Systems-II*, Vol. 51, No. 7, pp. 354-358, 2004.
- Chih-Min Lin and Ya-Fu Peng, "Missile guidance law design using adaptive cerebellar model articulation controller," *IEEE Trans. Neural Networks,* Vol. 16, No. 3, pp. 636-644, 2005.
- Chih-Min Lin and Chiu-Hsiung Chen, "Robust fault-tolerant control for biped robot using recurrent cerebellar model articulation controller," *IEEE Trans. Systems, Man, and Cybernetics, Part B,* Vol. 37, No. 1, pp. 110-123, 2007.

Thank You for Attention